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Statistical Stability Assessment

RATIONALE

Engine stability analysis and assessment, and the allocation of stability margins, form integral parts of the design and development of every gas turbine engine program. The existing guidelines for stability assessments are documented in ARP1420 and AIR1419, which have been widely utilized throughout industry. However, the existing guidelines do not recognize the statistics of operational demands and can therefore potentially overemphasize low probability events. Applying statistical techniques allow an assessment of the probability of encountering a destabilizing event and thus enable a risk assessment to be made for inlet/engine compatibility. The application of such techniques form the basis of the methodology discussed in this Aerospace Information Report.

FOREWORD

Engine stability analysis and assessment, and the allocation of stability margins form integral parts of the design and development of every gas turbine engine program. The existing guidelines for stability assessments are documented in ARP1420 and AIR1419, which have been widely utilized throughout industry. A key aspect of the stability assessment process involves assessing the statistical aspects of stability and the probabilistic nature of stability limits. As a result, there has been significant progress in developing and applying Monte Carlo techniques to the stability assessment process. A discussion of such techniques forms the basis of the methodology discussed in this Aerospace Information Report.

The overall approach outlined in the document is referred to as the Statistical Stability Assessment or SSA. Information is provided on the various models and methods that make up the SSA process. The validity and application of the approach is examined.

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1. SCOPE

This SAE Aerospace Information Report (AIR) provides a methodology for performing a statistical assessment of gas-turbine-engine stability-margin usage. Consideration is given to vehicle usage, fleet size, and environment to provide insight into the probability of encountering an in-service engine stall event. Current industry practices, such as ARP1420, supplemented by AIR1419, and engine thermodynamic models, are used to determine and quantify the contribution of individual stability threats. The statistical technique adopted by the S-16 committee for performing a statistical stability assessment is the Monte Carlo method (see Applicable References 1 and 2). While other techniques may be suitable, their application is beyond the scope of this document. The intent of the document is to present a methodology and process to construct a statistical-stability-assessment model for use on a specific system and its mission or application.

1.1 Purpose

The purpose of the Statistical Stability Assessment (SSA) is to provide a quantitative tool for optimizing engine stability margin requirements and provide greater visibility of off-design conditions. It can be applied at the inception of a new project to perform trade studies and support risk assessments; or to a mature project to assess the impact/risk of modifications or changes in operational usage.

This document provides guidelines on the procedures and data that are required to perform the statistical stability assessment. A number of illustrative examples are presented.

1.2 Field of Application

The compatibility of a gas turbine engine with its operating environment is a major concern in all gas turbine applications. The installation of an engine, be it in an aircraft, ship, or ground facility, determines the quality of airflow provided to the compression system of the engine and has a significant influence in the setting of the engine stability margin. This has a direct impact on the performance, operability, and mechanical integrity of the installed engine. It is the operability aspects that the SAE S-16 committee has devoted its main effort: understanding and accounting for the effects of intake flow-field distortion on engine stability.

The current industry guideline for assessing the effect of intake flow-field distortions has been the subject of previous SAE publications, notably ARP1420 and AIR1419. The loss of compression-system stability margin due to inlet flow distortion is combined with the losses from other factors to produce a stability margin stack-up. Traditionally, stack-ups are performed at a number of flight conditions that are judged to be the most critical by the engine company, based on information available from the airframe company and the operator. Destabilizing factors are judged to be either random or non-random and summed accordingly to provide an estimate of the likelihood of encountering engine instability at the chosen flight condition. In practice, stability margins are usually set such that engine instability is predicted not to occur.

While this methodology continues to serve the industry well, it is based on the assumption that the random factors are normally distributed and also offers limited insight into the likely stall rate for a fleet of aircraft. Further, the growing number of factors that are accounted for in the stability stack-up, the increased use of variable geometry, and advanced control modes have made it potentially more difficult to identify a small number of critical conditions that would indicate the engine to be truly stall free.

This document presents a statistical approach to model the engine operating environment with a view to identifying critical operating points and predicting a probability of compression system instability. Consideration is given to atmospheric variations, vehicle usage, and installation effects. The resulting stability assessment can provide improved insight into engine stability and be used to support trade studies, risk assessments, and design optimization.

To adequately capture the engine operating environment a detailed knowledge of the aircraft utilization is required. This extension of the stability assessment, therefore, demands much greater participation from both the operator and the airframe designer. The technique has its genesis rooted in military fighter applications, but is equally applicable to commercial aircraft/propulsion systems, helicopters, business aircraft, auxiliary power units, and other gas turbine installations.

As discussed previously, the Monte Carlo method was selected as the statistical tool because of its inherent ease of use and flexibility in handling a large number of variables with various distribution types that can be treated either as statistically independent or, if required, as having a statistical dependency.

2. REFERENCES

2.1 Applicable Documents

The following publications form a part of this document to the extent specified herein. The latest issue of SAE publications shall apply. The applicable issue of other publications shall be the issue in effect on the date of the purchase order. In the event of conflict between the text of this document and references cited herein, the text of this document takes precedence. Nothing in this document, however, supersedes applicable laws and regulations unless a specific exemption has been obtained.

2.1.1 SAE Publications

Available from SAE International, 400 Commonwealth Drive, Warrendale, PA 15096-0001, Tel: 877-606-7323 (inside USA and Canada) or 724-776-4970 (outside USA), www.sae.org.

ARP210	Definition of Commonly Used Day Types (Atmospheric Ambient Temperature Characteristics Versus Pressure Altitute); Supersedese MIL-HDBK-310 and MIL-STD-210A
AS681	Gas Turbine Engine Steady-State and Transient Performance Presentation for Digital Computer Programs
AIR1419	Inlet Total-Pressure-Distortion Considerations for Gas-Turbine Engines
ARP1420	Gas Turbine Engine Inlet Flow Distortion Guidelines
AIR5866	An Assessment of Planar Waves
ARD50015	A Current Assessment of the Inlet/Engine Temperature Distortion Problem

2.1.2 ASME Publications

Available from ASME, 22 Law Drive, P.O. Box 2900, Fairfield, NJ 07007-2900, Tel: 973-882-1170, www.asme.org.

ASME 97-GT-48	Some Applications of the Monte Carlo Method to Gas Turbine Performance Simulations: Krurzke & Joachim
ASME 91-GT-402	A Global Approach in Evaluating Inlet/Engine Compatibility: Csavina & Denney
Transactions of ASME	Journal of Engineering for Gas Turbines and Power, Aircraft Engine Control Mode Analysis, pp. 838-844: Brown & Elgin

2.1.3 U.S. Government Publications

Available from the Document Automation and Production Service (DAPS), Building 4/D, 700 Robbins Avenue, Philadelphia, PA 19111-5094, Tel: 215-697-6257, http://assist.daps.dla.mil/quicksearch/.

MIL-HDBK-310	Global Climatic Data for Developing Military Products (Superseded by SAE ARP210)
MIL-STD-210A	Global Climatic Data for Developing Military Products (Superseded by MIL-HDBK-310 then SAE ARP210)

2.2 Applicable References

1. Rubinstein, R.Y., Simulation and the Monte Carlo Method, John Wiley and Sons, New York, 1981.

2. Hammersley, J. M., and D. C. Handscomb, Monte Carlo Methods, Methuen, London, 1964.

3. Knuth, The Art of Computer Programming, Vol. 2: Seminumerical Algorithms, Addison-Wesley, 1991.

4. Keppler, Random Numbers Made Simply, Computer Language Magazine, June 1993.

5. Press, et al., Numerical Recipes: The Art of Scientific Computing, Cambridge University Press, 1989.

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7. Swan, Heads I Win, Tails You Lose, Dr. Dobb's Journal, December 1993.

8. Simon, G., Applied Statistics, 1976, pp.266-274.

9. Kotz, Samuel, and N. L. Johnson, editors, Encyclopedia of Statistical Sciences, Vol. 5, John Wiley and Sons, New York, 1985.

10. Hartley, H. O., Statistical Methods for Digital Computers, John Wiley and Sons, New York, 1977, pp.16-34.

2.3 Symbols and Abbreviations

Alpha, alpha or AOA	Angle of Attack
ALT or alt	Altitude
APU	Auxiliary Power Unit
BLD	Compression system bleed
Beta, beta or AOSS	Angle of Sideslip
Cd	Nozzle Discharge Coefficient
CFD	Computational Fluid Dynamics
Cf	Nozzle Flow Coefficient
CIP	Component Improvement Program
CPU	Central Processing Unit
DIC	Circumferential Distortion Indices
DIRT	Tip Radial Distortion Indices
DIRH	Hub Distortion Indices
DPRA	Change in Available Pressure Ratio
DPRO	Change in operating line pressure ratio

DPRS	Change in stability pressure ratio
DSM	Change in stability margin
EFH	Engine Flight Hours
FOD	Foreign Object Damage
GETUP	Generalized Engine Throttle Usage Prediction
HPC	High Pressure Compressor
HPX	Horsepower Off-take
HR	Hours
Ks	Probability factor
LCG	Linear Congruential Generator
M or Mn	Mach Number
Max or MAX	Maximum
MC	Monte Carlo
MES	Main Engine Start
N	Weibull distribution coefficient
$N/\sqrt{\theta}$	Corrected Rotational Speed
NPR	Nozzle Pressure Ratio
PLA	Power Lever Angle
PR	Component Pressure Ratio
PRO	Operating line Pressure Ratio
PRS	Stability limit line Pressure Ratio
P^2/W	Combustor Loading Parameter
P/P	Component Pressure Ratio
RSS	Root Sum Square
REC	Inlet pressure recovery
SFC	Specific Fuel Consumption
SM	Stability Margin
Smavl or SM AVAIL	Available Stability Margin

SSA	Statistical Stability Assessment
TAC	Total Accumulated Cycles, where a cycle represents a defined sequence of engine throttle transients
TRANS	Transient
$\Delta T/T$	Temperature Ratio
$W\sqrt{\theta}/\delta$	Corrected Airflow
a	Weibull distribution coefficient
b	Weibull distribution coefficient
f/a	Fuel Air Ratio
g	Acceleration due to gravity
m	Probability distribution central moment, mean
s	Probability distribution central moment, standard deviation
Δ	Delta change
η	Efficiency
σ	Standard deviation
σ_{NEW}	Standard deviation of new engine performance
σ_{OH}	Standard deviation of overhauled engine performance

3. DESCRIPTION OF MONTE CARLO METHODOLOGY

Monte Carlo methodology is a modeling technique that is widely used throughout engineering, science, and commerce. It enables the overall statistical variability of a process to be determined, based on the known variability of the process inputs. Statistical distributions are used to define the variability of the inputs to the process, and these are sampled randomly many times to provide information about the overall process output. The input parameters to the process are defined at a level such that their distribution can be readily defined or empirically determined. These distributions can take many forms, ranging from uniform or normal distributions to non-uniform discrete distributions that capture the true variation of an input parameter. To create a statistically significant database, the method requires that the individual inputs are randomly sampled a sufficiently large number of times. To achieve this, a computer simulation of the process is generated and a Random Number Generator used to sample each input distribution many times. The complexity of the simulation depends on the process being modeled. The flexibility of the technique is such that the simulation can be extended to large systems and easily tailored to include input parameters that are statistically interdependent as well as those that are statistically independent. The technique facilitates parametric analyses within the model, establishing output sensitivities due to changes in component parameters.

A simple example of a Monte Carlo simulation is shown in Figure 1. A process is represented wherein two washers, from different sources, are to be fitted to a unit and an understanding of the total variation in thickness is required. Washer A is produced from a process that yields much greater variability than that for washer B. The cumulative probability of the thickness for each washer is shown along with its probability density function and this is sampled repeatedly using uniformly-distributed random numbers. The overall thickness of each combination of washers is then calculated. By considering a sufficiently large number of samples, the resulting distribution of combined washer thickness is derived.

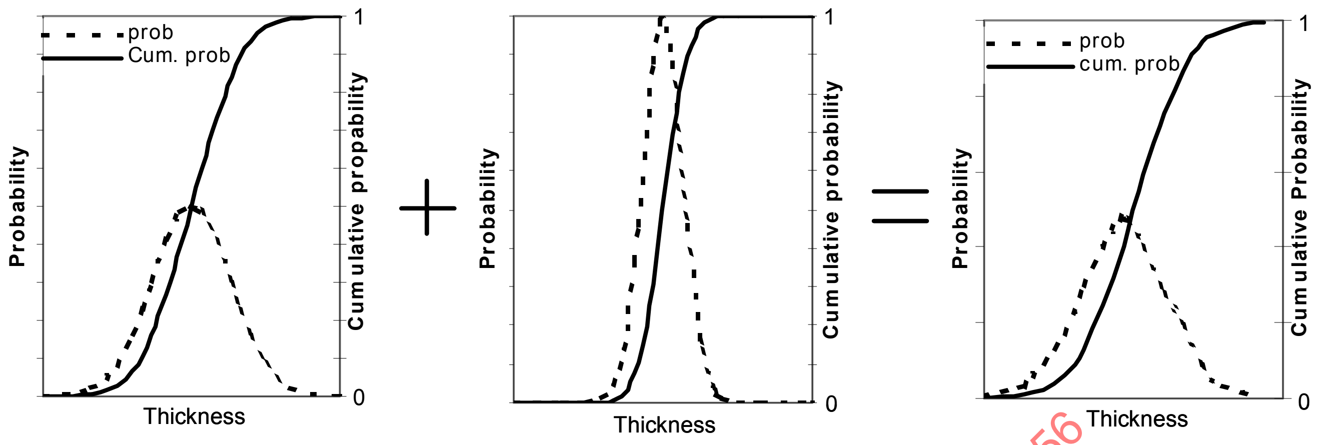


FIGURE 1 - EXAMPLE OF MONTE CARLO SIMULATION

In this example, simple normal distributions were assumed for two independent variables.

The Monte Carlo simulation is applied generally to very much more complex problems where there are many variables, each with their own statistical distribution, the variables being either statistically independent or statistically dependent. Where the value of a variable is dependent on the value of another variable, their values can be selected using the same random number or there may be a dependency such that a given value of a variable will lead to a new constraint on another variable. Such relationships between variables can be programmed into the Monte Carlo simulation.

For complex processes, the logical sequence of events or components of the process need to be determined and used to develop the Monte Carlo simulation. The simulation will then interrogate the variable distributions in turn and [should] reflect the actual process. An example process is shown in Figure 2. In this example, the process output, D, is a function of variables A, B and C. A and B are statistically independent while the value of C is a function of B and has a statistical distribution. The simulation is required to interrogate model C with a value of B and a random number in order to obtain a value of C.

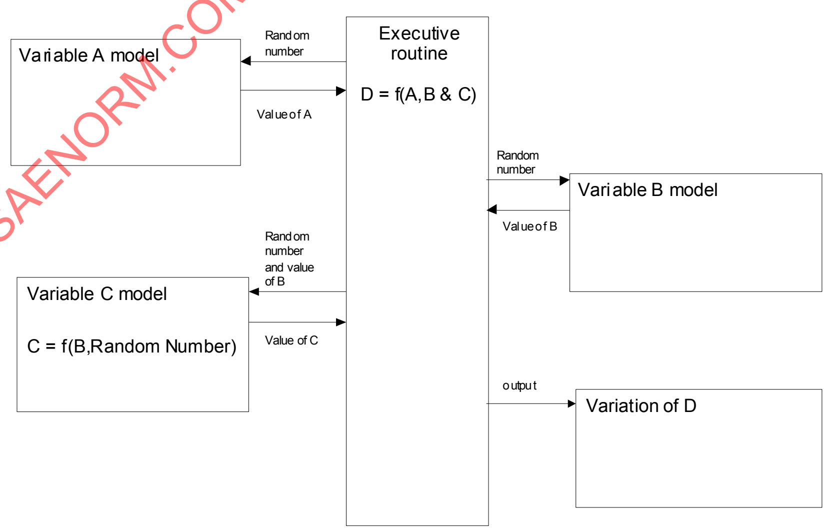


FIGURE 2 - EXAMPLE OF MONTE CARLO SIMULATION PROCESS

The routine that forms the basis of the simulation has been termed the ‘executive routine.’ Additional functionality can be built into this routine to give the simulation flexibility. For example, being able to assign given variables a fixed value enables sensitivity studies of specific variables to be performed. It also enables parametric studies to be carried out.

Two important considerations when developing a Monte Carlo simulation are the sample size and the choice of random number generator. The appropriate sample size is usually optimized during the development of the model. A sufficiently large number of samples is required to ensure adequate repeatability of the output but with a manageable simulation run time, as discussed in 6.2.2. The random number generator must be capable of producing a distribution of numbers appropriate to the sample size being considered. A rule of thumb is that the period of the generator be at least twice the size of the number of samples required by the application. Some random number generators enable the user to specify a seed such that a repeatable series of random numbers is generated. This feature is useful for debugging the application or for performing some sensitivity studies. These issues are discussed further in Appendix A.

The flexibility of the Monte Carlo simulation makes it an ideal technique for assessing complex statistical problems such as gas-turbine-engine stability-margin assessment. It should be noted that the accuracy of the simulation output depends on how well the process elements and their distributions are modeled.

4. STATISTICAL STABILITY ASSESSMENT (SSA)

The factors that are considered in a stability assessment are shown in Figure 3 (although this is not an exhaustive list). Historically, a simple arithmetic summation of all factors was performed to arrive at the required compression-system stability margin at a given flight condition. This historic practice was defined as the ‘worst case’ stability margin stack-up. Subsequently, as defined in Appendix A of AIR1419, it was recognized that the probability of all worst-case values occurring simultaneously was low and that some of the factors should be summed statistically. The factors defined as being random in nature included items such as engine-to-engine variation. These were assumed to have normal distributions and their effects on stall margin were summed using a root-sum-square method. This form of stability assessment has become the recognized current practice used throughout industry. While having a statistical element, the methodology does not recognize factors with non-uniform distributions or provide insight into the probability of instability in service. Further discussion of this practice is presented in Appendix B.

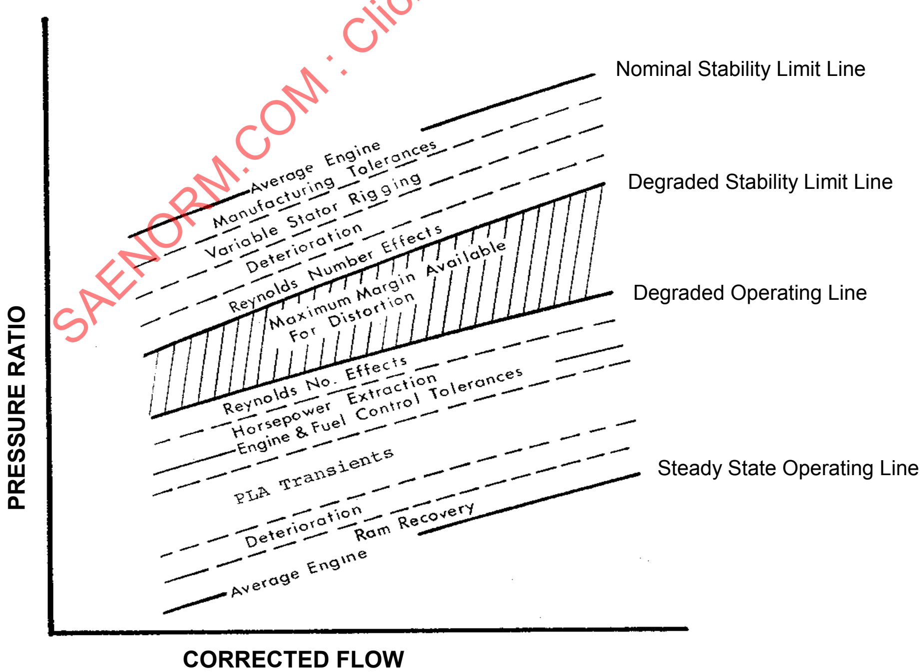


FIGURE 3 - TYPICAL COMPRESSOR DESTABILIZING FACTORS

In addition, the increasing complexity of engine control systems and variable geometry are such that it becomes difficult to select a small number of worst-case scenarios that, if shown to have positive residual stability margin, would indicate the engine to be stall-free. The statistical stability assessment methodology has been developed as an extension to the current stability audit to enable parameters with non-normal distributions to be appropriately considered and to give insight into stability-margin usage and potential stall rate. The use of the Monte Carlo technique enables very complex installations and processes to be modeled allowing the simulation to be used as a design tool to identify critical flight conditions.

It is important to note that the existing stability assessment method of AIR1419 can be a sub-set of the SSA methodology. This can be accomplished by including a default option in the SSA model that duplicates the existing stability assessment.

The statistical assessment requires the physics underlying the operational conditions and the vehicle characteristics to be input with the variability and distributions that reflect the system usage. The methodology accomplishes this by modeling the significant elements that influence the stability of the compression-system components, incorporating the independent and dependent variability of the elements, as appropriate.

There are many factors that influence the stability of the compression system of the engine. For the purpose of simulating operation in an aircraft, they can be divided into four major groups. These groups are the environment, the physical installation, the usage, i.e. the operational condition, and the engine. These groupings are thought to be fairly generic, but can be tailored or changed for different gas-turbine applications, as necessary. The Statistical Stability Assessment requires a model for each of these groups to be produced such that, when interrogated, they will simulate the overall operational experience.

4.1 System Architecture

The basic architecture for a Monte Carlo simulation was described in Section 3. The architecture required for a statistical stability margin assessment is shown in Figure 4. The executive routine interrogates the models in the logical order to define a given flight condition. First, the environmental model is interrogated to set the ambient conditions. Second, the usage model provides specific data such as altitude, Mach number, aircraft attitude, and power lever angle (PLA) position. Then, these data are used as input into the aircraft model to determine engine-entry-flow parameters. Having determined the relevant data needed to describe a flight condition, an engine model is used to assess stability margin usage. The engine model combines the effects of the external influences with the internal engine influences.

4.2 Executive Routine

The executive routine serves to control the flow of data to and from the models, and controls the interrelationships between parameters. The executive routine produces a random number for each variable using a suitable random number generator. There are several types of random number generators. Important considerations when choosing a generator are discussed in Appendix A. The random number is supplied to the model along with the relevant dependency data. For example, a call for an angle of sideslip will require not only a random number, but also details of Mach Number, altitude, and angle of attack. Based on the information supplied, each model returns a parameter value from its associated probability distribution. This process continues until a complete definition of a flight point is produced and a stability margin usage obtained. In practice each model will be utilized several times during each pass through the simulation. This is because some variables will be dependent on others from the same model. There are also some cases when the simulation may need to address the models out of the simple logical sequence or even perform a number of iterations. An example of this would be the calculation of intake pressure recovery and distortion that are dependent on the engine flow. The executive routine must be organized to manage such cases efficiently and minimize computer time. More examples of parameter dependencies are provided in Appendix C.

The executive routine can accommodate varying degrees of complexity. It can be a simple simulation of a particular problem or it can be a complex interactive computer program that provides the user with many options for tailoring the analysis. In its most complex form, it should be possible to assign a fixed value to any variable, to change parameter dependencies, and to offer options for different outputs. The more flexible the simulation, the more useful it will become for enabling parametric trade studies to be performed and "what-if" type assessments to be made. An executive routine could be programmed fairly simply in any computer language. Alternatively, there are commercially available codes that can serve as the executive routine by integrating the various models and performing the Monte Carlo function.

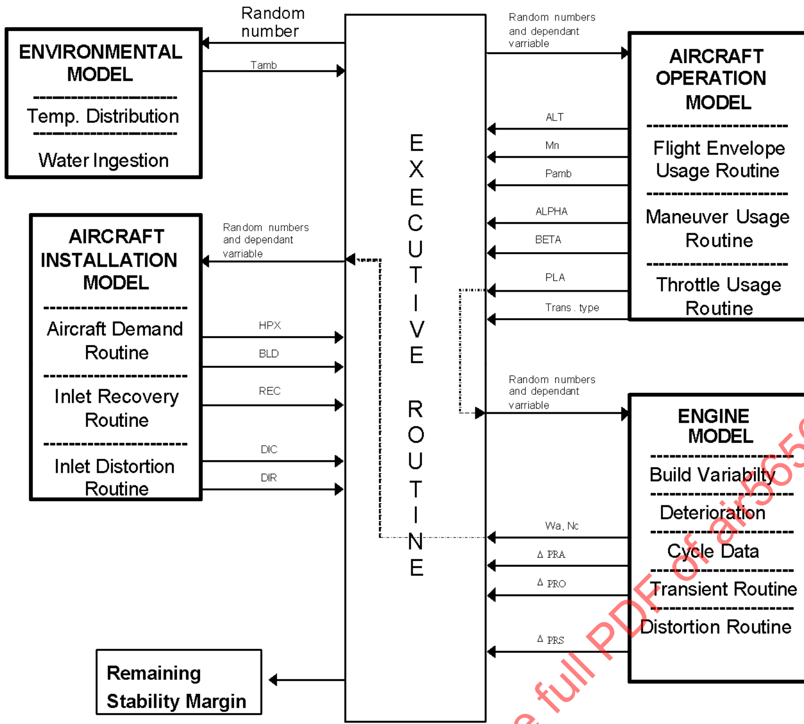


FIGURE 4 - AIRCRAFT/ENGINE SYSTEM ARCHITECTURE

4.3 Simulation Models

For the purpose of simulating a gas turbine engine in an aircraft, four basic groups of data have been identified. It is recommended that each group be considered as a particular model, containing a number of variables. As with the executive routine, the complexity with which variables are modeled depends on the use of the simulation, but the better the data, the more meaningful the simulation will be.

The four models are:

- Environmental model to define the variability of the atmospheric conditions. This model should contain parameters such as temperature, pressure, and humidity.

- Usage model to define the operation of the aircraft including capabilities, missions, and pilot operating characteristics. The model is used to determine values of altitude, Mach number, AOA, AOSS, power lever angle and engine transient type.

- Aircraft model to define values of inlet recovery, distortion, horsepower off-take, and customer bleed. This model could be extended to consider variability of these items.

- Engine model that defines the engine physical and functional characteristics such as, build tolerances, deterioration, and other variables, and how the engine operates in or responds to the given flight condition. This model should be capable of providing stability margin used and stability margin remaining back to the executive routine to form part of the output data. Both steady-state and engine transient simulations are used in this model.

Greater detail of the data that should be included in each model is contained in Section 5.

An appropriate statistical distribution needs to be defined for the applicable variables contained within the models. The normal distribution is an obvious choice for modeling variables with a truly random element, such as build scatter, while a discrete distribution is more appropriate for variables such as flight-envelope usage or service demands. A brief description of some useful distributions for the modeling of factors that influence engine stability margin is given in Appendix D.

4.4 Model Execution

The modeling of a complex system, such as an aircraft and its propulsion system, can involve a great number of variables. When performing the Monte Carlo simulation it is important that a sufficient number of passes be considered. During the development of a Monte Carlo simulation model, cases should run with a range of passes to find the optimum number of passes required for convergence. During this exercise, a careful watch should be kept on the random number generator as it may begin to repeat after a given number of passes and hence, invalidate the randomness of the simulation. Each pass through the simulation is considered as a sample. The user should be very careful to avoid the pitfall of using too few passes when the solution would not converge to an acceptable accuracy. Guidelines for ensuring that this is avoided are provided in 6.2.2.

5. DESCRIPTION OF SIMULATION MODELS

This section gives examples of how the individual models can be constructed. Although the following models and their subroutines are representative of a modern-day fighter application, they are indicative of what would be required to simulate any gas turbine application.

5.1 Environmental Model

The environmental model, shown in Figure 5, describes the ambient air temperature (at altitude) and other weather conditions, which are to be considered for the flight envelope defined by the usage model. This model depicts the demands on the engine due to environmental conditions. The most obvious is ambient temperature, as it affects the corrected rotor speeds and airflow and, thereby, the fan and compressor match points. Other factors can be included such as water ingestion from heavy rainfall. Statistical data on worldwide weather conditions are readily available from a number of sources. A typical example is MIL-HDBK-310. Such environmental models may be easily incorporated into a Monte Carlo simulation. Engine synthesis models already accept ambient temperature as an input variable, thereby thermodynamically responding to temperature variations. These temperature variations affect clean available stability margin due to fan and compressor match point changes.

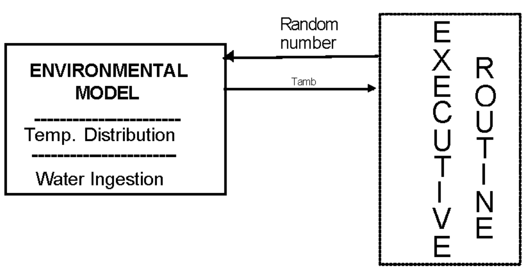


FIGURE 5 - ENVIRONMENTAL MODEL

If results are desired for variable temperature as opposed to using specific “days” (i.e., hot, cold, std), a variable temperature distribution can be found in MIL-HDBK-310, which gives the cumulative probability of temperature variation relative to Standard Day temperatures. This distribution gives more extreme temperatures than the cold and hot day atmospheres, but the probabilities are very low for these extremes.

Theater-specific environmental models can also be produced. In this case, environmental data can be obtained from the closest weather recording post. An example of a theatre-specific ambient temperature distribution is shown in Figure 6, compared to that for a standard day. It shows there is a significant shift (approximately 35 °F cooler) in the distribution for Eielson AFB Alaska, which would keep corrected flows higher at their scheduled military/maximum power settings, thereby generating maximum distortion levels at a much higher probability than for a standard atmosphere. Fan and compressor rotor tip clearances would also be affected especially at take-off conditions, where rotors would be colder because they have yet to be thermally stabilized at power.

A relevant rare-condition-driven stall event (although important) would be one destabilized by extreme water ingestion. In this case, the cycle deck must have inputs for water to air ratio and be able to respond (thermodynamically and aerodynamically) to show the impact on both fan and compressor stability margins. The ability to accurately model rain ingestion is a significant challenge for cycle models. Theater-specific statistical weather data can be found by referring to the FAA weather office or the National Climatic Data Center. In the case of the U.S. Air Force, a central information source is available at the Wright-Patterson Air Force Base Weather Office.

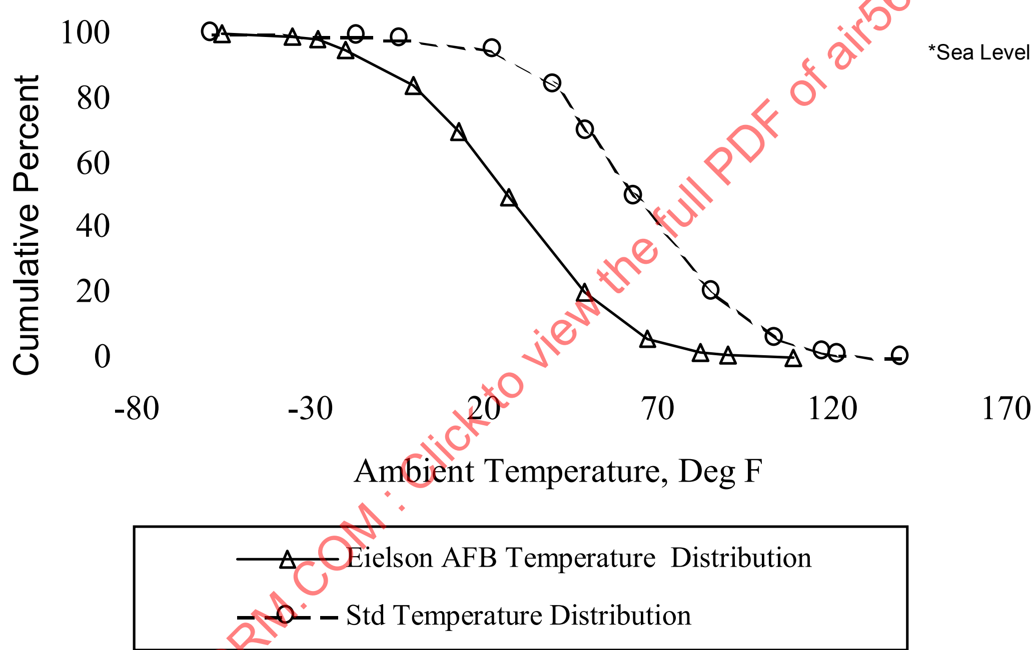


FIGURE 6 - THEATRE-SPECIFIC AMBIENT TEMPERATURE DISTRIBUTION

5.2 Aircraft Operations Model

The Aircraft Operations Model describes the mission or operational conditions to be analyzed in terms of altitude, Mach number, angle of attack and sideslip, engine throttle position (PLA), and type of throttle transient. An aircraft operations model diagram is shown in Figure 7.

For the aircraft operations model, which depicts where and how the aircraft is flown, historical data from flight recorders, and flight simulator data flown to representative missions, if available, are most appropriate for this application. However, in some cases estimated data are required.

The aircraft operations model consists of three routines that are discussed from a fighter aircraft perspective. They are the Flight Envelope Usage Routine, the Maneuver Envelope Usage Routine, and the Throttle Usage Routine.

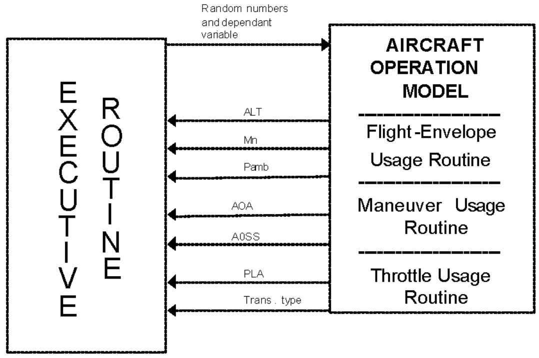


FIGURE 7 - AIRCRAFT OPERATIONS MODEL DIAGRAM

5.2.1 Flight-Envelope Usage Routine

Since this section on flight-envelope usage is one of the first discussions of the technique for development of probability distributions to be used in this overall Monte Carlo simulation, greater detail will be presented in this section than in the sections that follow.

Mach/altitude usage data from onboard recorders from a fleet of modern military fighters are shown in Figure 8. The time in hours per 1000 hours and the number of transfers per 1000 hours into and out of each Mach/altitude block are shown. The data were not available in more detail than is shown, and it is assumed the usage is distributed uniformly within each block.

It should be noted that the number of blocks should be chosen such that there is not a significant variation of residence time across a block. While a large block was used to cover the top left-hand corner of the flight envelope, it should be noted that this would result in overexposing the engine to areas of minimal stall margin. A more representative approach may be to represent this region with a number of smaller blocks that would more accurately represent actual usage.

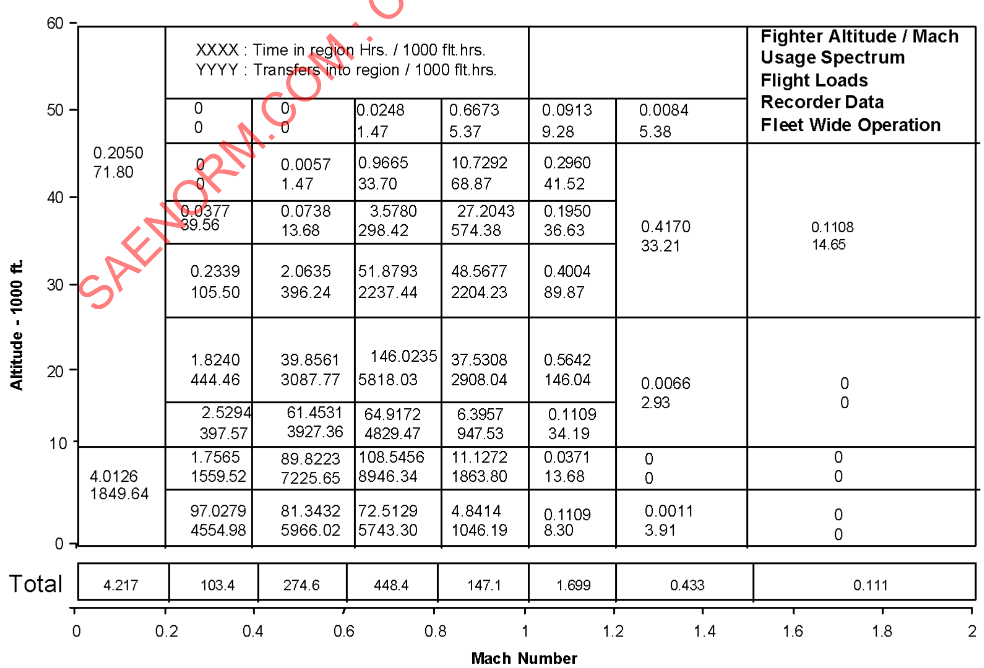


FIGURE 8 - TYPICAL FLIGHT-ENVELOPE USAGE DATA

In order to develop a statistical model, the data must be characterized as a function of both altitude and Mach number. The Mach number distribution will be addressed first.

The summation of time in each Mach number block is shown at the bottom of Figure 8. The data from this summation were converted to percentages of total time and used to construct the discrete probability distribution as a function of Mach number, as shown in Figure 9. This approach is consistent with the assumption that the data are uniform within the original blocks. These same data were then converted to the equivalent cumulative probability distribution shown in Figure 10. The cumulative probability distribution is the form of the data that is used in the Monte Carlo simulation. A random number between 0% and 100% is used to obtain a Mach number from the cumulative probability distribution shown in Figure 10; this Mach number is then used for a given pass through the overall simulation. Since the random numbers are uniformly distributed, the derived Mach numbers for various passes through the simulation will be distributed as in the original curve.

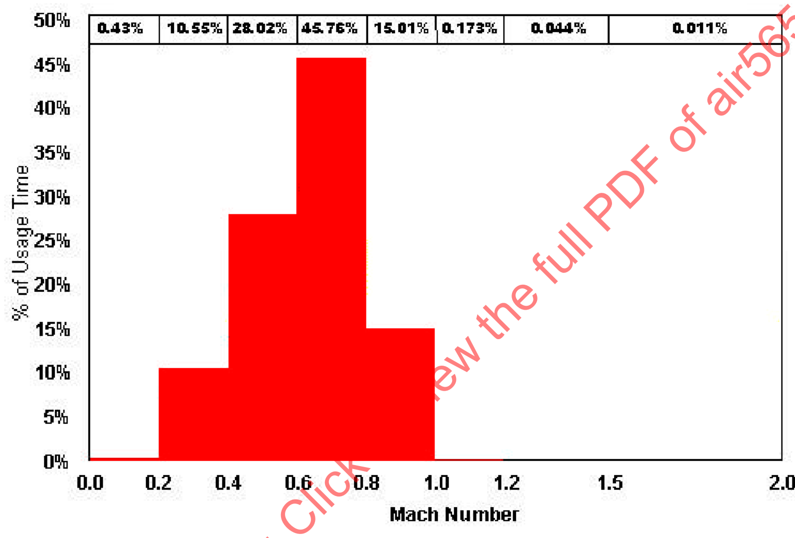


FIGURE 9 - MACH NUMBER PROBABILITY DISTRIBUTION

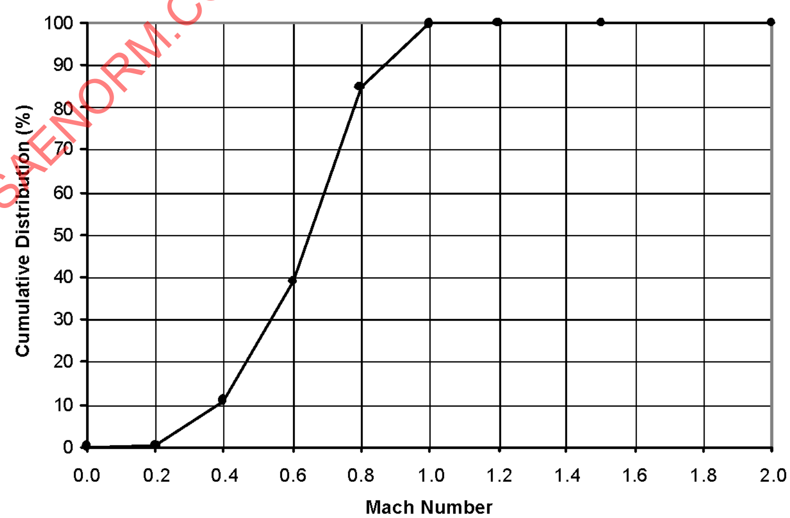


FIGURE 10 - MACH NUMBER CUMULATIVE PROBABILITY DISTRIBUTION

To demonstrate that the original data are reproduced to a high degree of fidelity, the data of the cumulative distribution (Figure 10) were used in a Monte Carlo simulation and the resulting Mach number probability distribution is compared to the original distribution in Figure 11.

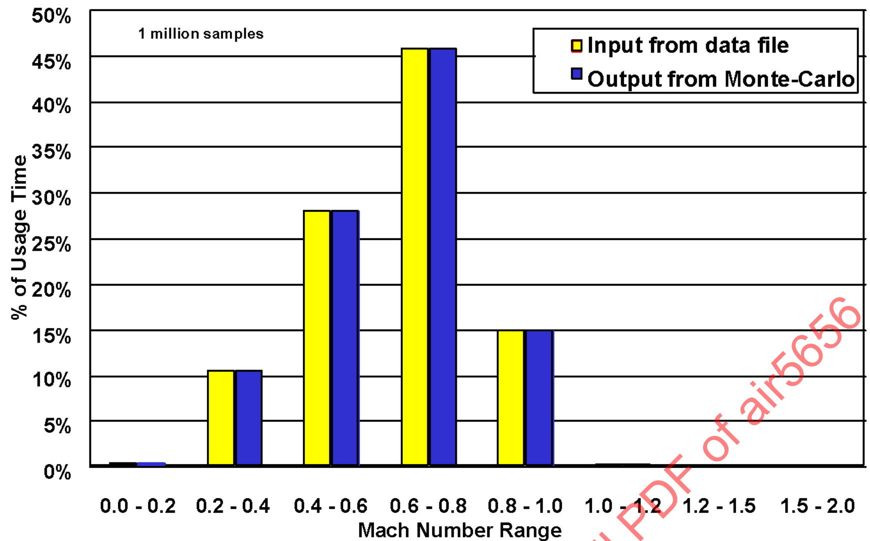


FIGURE 11 - COMPARISON OF MONTE CARLO SIMULATION DATA

The altitude distributions for each of the Mach number blocks were obtained in a similar manner. It was desired to break the altitudes into increments of 5000 feet. In several cases, the original data are for blocks with a larger altitude range. Again, it was assumed that the data were distributed uniformly within the original blocks and the distributions were constructed accordingly. The resulting cumulative probability distributions for each of the Mach number blocks are presented in Figure 12.

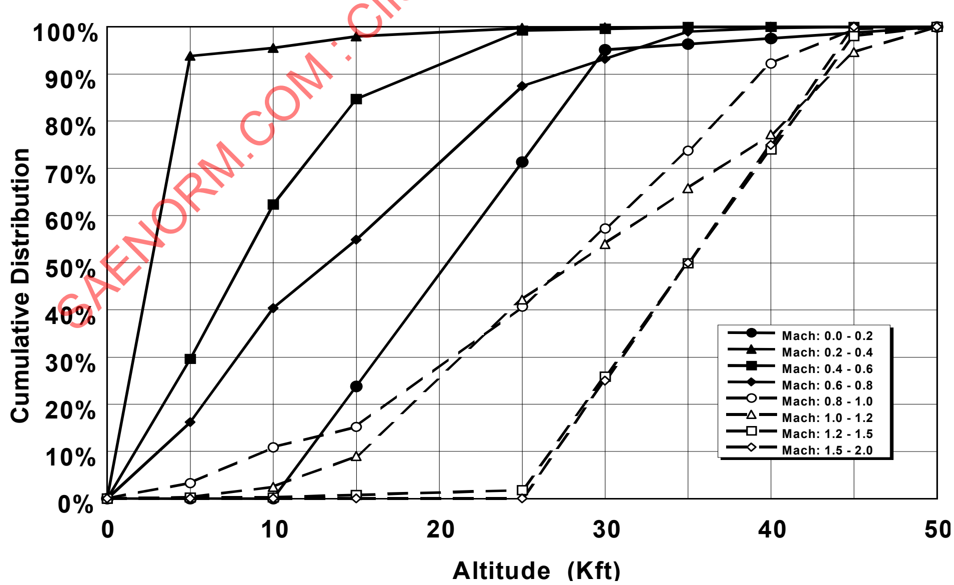


FIGURE 12 - ALTITUDE CUMULATIVE PROBABILITY DISTRIBUTIONS

Again, to demonstrate that the original distribution is reproduced to a high degree of fidelity when used in a Monte Carlo simulation, the resulting probability distribution is compared to the original distribution (for one Mach number block) in Figure 13.

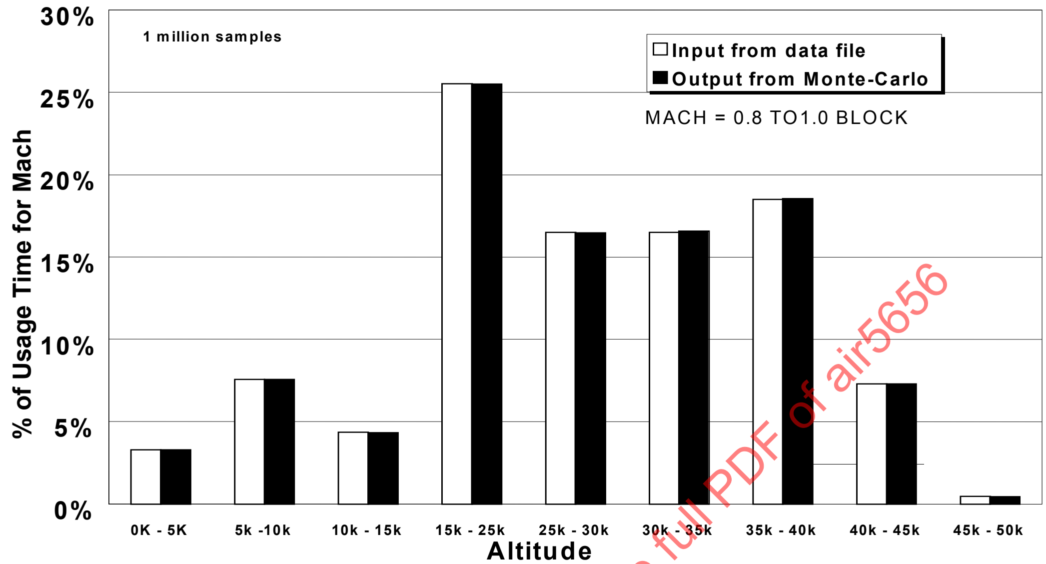


FIGURE 13 - COMPARISON OF MONTE CARLO SIMULATION DATA

Using the statistical usage data and two random numbers, a Mach number and altitude can be generated for use in subsequent steps of the Monte Carlo simulation.

5.2.2 Maneuver Envelope Usage Routine

Angle of attack and sideslip conditions are usually restricted within the aircraft design maneuver limits for a given Mach number. A representative maneuver envelope is shown in Figure 14. The frequency of occurrence of each AOA/AOSS condition is distributed within the maneuver limits. Thus, the distribution is truncated at the AOA/AOSS design limits. Aircraft loads data from flight recorders or simulator outputs can also be a source to obtain the AOA/AOSS distributions. Departed flight may also be modeled, if desired. In this example, the frequency of occurrence of each AOA/AOSS condition is assumed to follow normal distributions truncated at the maneuver limits. Thus, both AOA and AOSS are dependent on Mach number, and AOSS is also dependent on AOA.

Another consideration that should be included in the AOA/AOSS selection is the limitation on angle of attack and sideslip imposed by maximum “g” capability of the aircraft. Within the ranges of these values that are considered here, limiting AOA and AOSS are inversely proportional to the dynamic pressure. For example, if at a given Mach number, the angle of attack is limited to 30 degrees at 30kft altitude, then the maximum AOA is limited to 10 degrees at a lower altitude of 3200 feet where the dynamic pressure is three times the value at 30kft.

At this point Mach number, altitude, AOA, and AOSS have been determined for use in subsequent steps of the simulation.

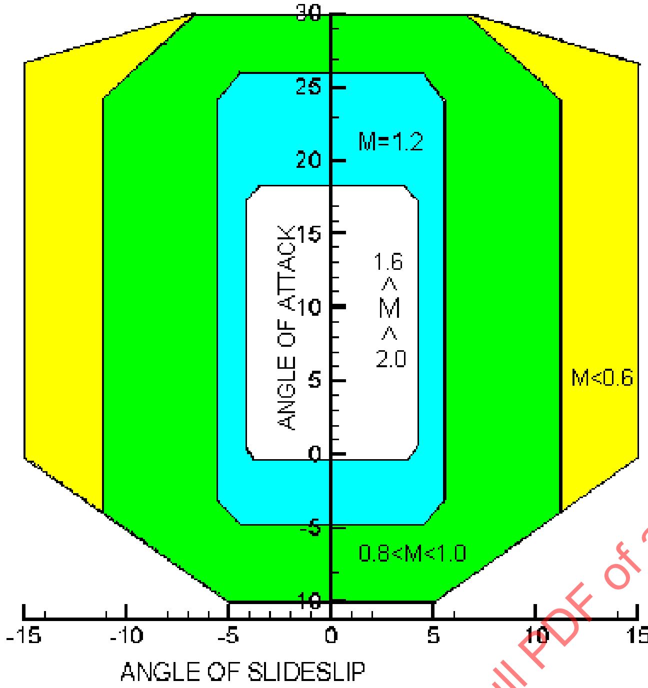


FIGURE 14 - MANEUVER ENVELOPE USAGE MODEL DIAGRAM

5.2.3 Throttle Usage Routine

Aircraft engine throttle usage data may be used to identify the types of destabilizing throttle transients that are typically performed, and to determine their frequency of occurrence and where in the flight envelope (altitude, Mach) they are encountered.

As an example, analysis of mission data gathered on a twin-engine fighter aircraft evidenced the following types of destabilizing transients:

- High power accelerations (rapid throttle movement from low or part power to near intermediate or above)
- Low power accelerations (from near idle to some higher power)
- Bode (from intermediate or above to part power and return in rapid succession)
- Deceleration (chop from near intermediate to near idle)
- Afterburner light
- Afterburner cancellation

In addition, fixed-throttle time and ground time prior to takeoff were taken into account. These transient types were chosen in order to capture known destabilizing effects. High and low power accelerations, Bodes, and afterburner cancellations are known to challenge the compressor, and decelerations and A/B lights are known to challenge the fan. Fixed-throttle operation may occur during an aircraft maneuver that generates high inlet distortion that may challenge either the fan or the compressor.

For analysis purposes, the throttle usage data was organized into two parts: (1) frequency of occurrence for each throttle transient type, and (2) the likelihood of encountering a particular throttle transient at any given altitude/Mach condition. In Table 1, the following frequencies were observed:

TABLE 1

Transient Type	Description	Occurrences per flight hour
1	High power accel	46.61(46 610/1000 EFH)
2	Low power accel	28.49(28 490/1000 EFH)
3	Bode	0.82(820/1000 EFH)
4	Decel	2.92(2 920/1000 EFH)
5a (5b)	A/B Light (cancellation)	18.17(18 170/1000 EFH)
6	Fixed-throttle	73.08(73 080/1000 EFH)

For this example, the above throttle transient types showing the percentage usage in various altitude/Mach bins are presented in Figure 15. The numbers in each block represent the percentage likelihood of encountering any particular transient types that are initiated in that altitude/Mach region. Similar outputs may be made for the other transient types.

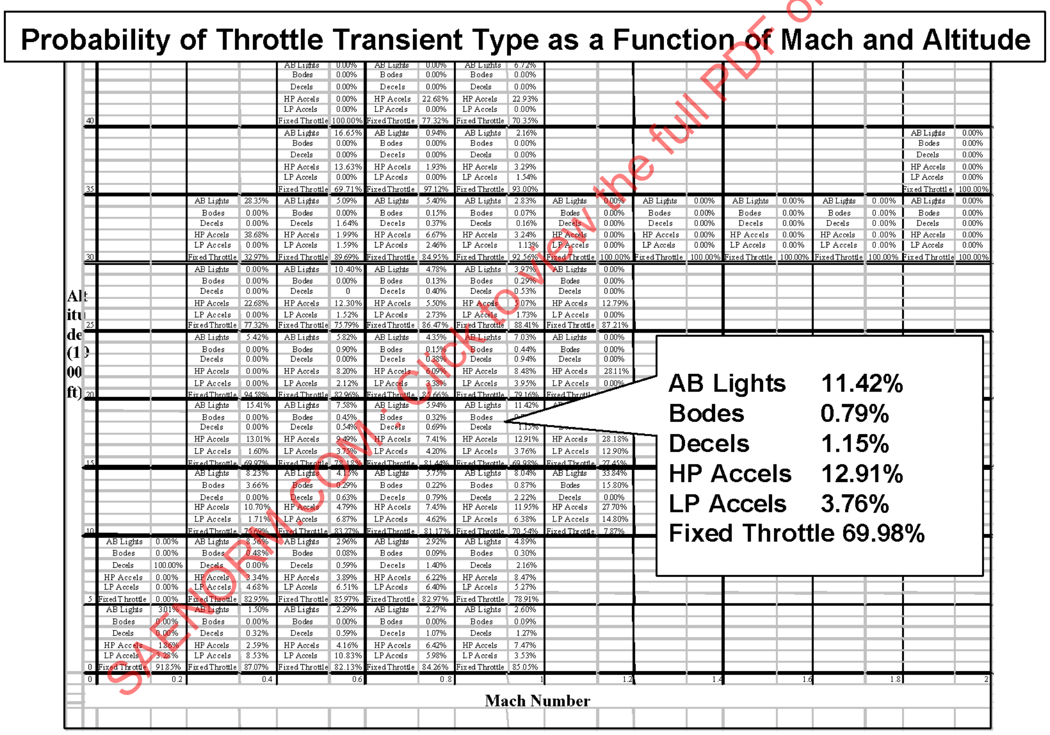


FIGURE 15 - TRANSIENT TYPE OCCURRENCES AS A FUNCTION OF ALTITUDE AND MACH NUMBER

For simulation purposes, this information can be used to determine the likelihood of encountering any particular throttle transient at the altitude/Mach condition selected by the flight-envelope usage routine (see 5.2.1).

Audit points are defined at a particular airflow in the transient where the minimum stability margin occurs. This point is referred to as the pinch point. The pinch point to be audited is dependent on the throttle transient type. For a fixed-throttle case, the airflow to be used is defined by the throttle setting. This may be intermediate or max power if the aircraft is in a climbing or turning maneuver, or may be the power for level flight if the aircraft is cruising. For the transient throttle cases, the correct airflow to audit needs to be determined. A gas generator transient from idle to intermediate with all the destabilizing elements book-kept throughout the entire airflow range is shown in Figure 16. For this particular compression component, the point at which minimum stability margin occurs is approximately 80% airflow. The correct airflow to accomplish the audit at would be at the "pinch point" (80% airflow). Each compression component is unique in its minimum stability margin or pinch point. It is dependent on the destabilizing factors such as the shape of the clean stability limit line, the inlet distortion and engine distortion sensitivities, and the acceleration fuel scheduling.

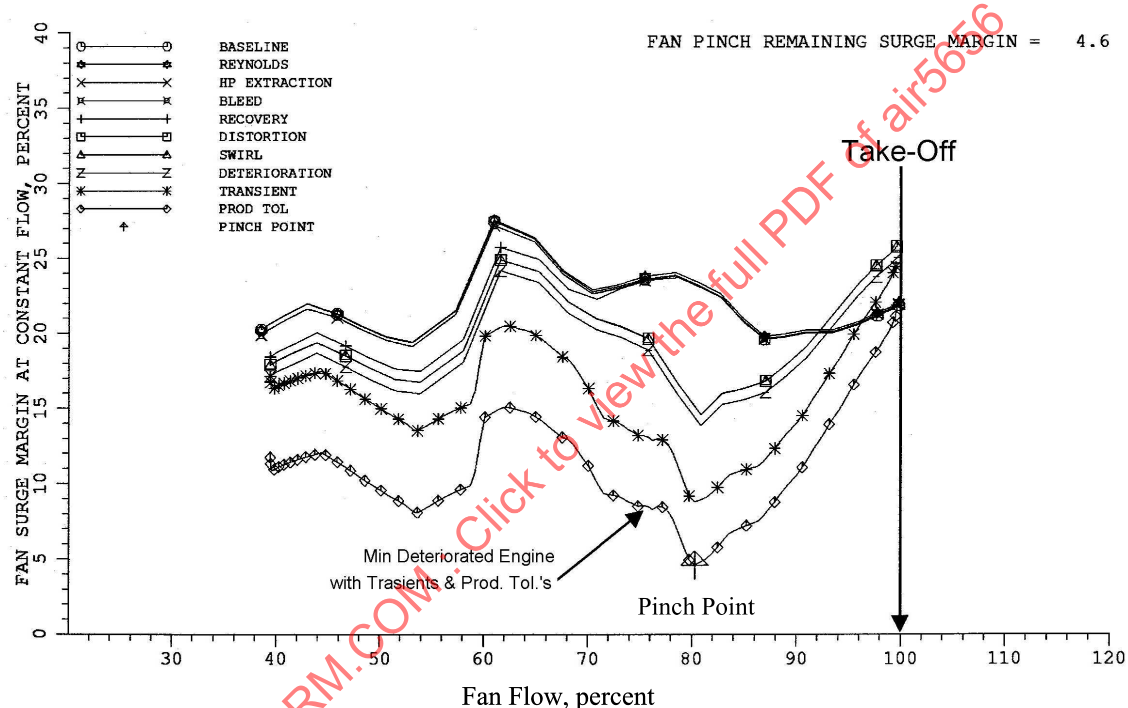


FIGURE 16 - SELECTION OF AUDIT PINCH POINT FOR TRANSIENTS

If actual mission data are not available, simulations may be used to develop engine throttle transient predictions. Data sources for flight condition thrust/drag dependency and mission dependency effects on throttle usage can come from a model such as the U.S. Navy's Generalized Engine Throttle Transient Usage Prediction (GETUP) code. GETUP is a three degree-of-freedom, point-mass flight simulation model that computes the aircraft flight path and engine throttle history, given a mission profile and the necessary engine and aircraft performance characteristics. Most aircraft manufacturers now have developed the ability to generate such data with their own computer programs. Such programs may be especially useful for state-of-the-art technology where historical data are not applicable.

5.3 Aircraft Installation Model

The Aircraft Installation Model specifies aircraft installation factors that also become inputs to the Engine Model. The prime item is the specification of aircraft inlet distortion in a form compatible with the distortion methodology of the engine manufacturer. Inlet pressure recovery characteristics are necessary in most cases for the determination of corrected airflow and absolute engine pressure levels. Also required is the specification of aircraft off-take levels such as engine bleed and horsepower off-take. A diagram showing the Aircraft Installation Model relative to the other models is shown in Figure 17. The input dependent variables (Mach number, AOA, AOSS, and corrected airflow) and outputs of the Aircraft Installation Model are also shown.

Pressure recovery and distortion data for the aircraft model is normally obtained through scale model testing. This model is normally deterministic in nature. However, if inlet-to-inlet variations are to be considered, distributions attributed to manufacturing variability (associated with throat area and inlet lip contour variations) may be included in the correlation. Data for engine off-takes are generally estimated from past experience or a specific power summation for the applicable vehicle.

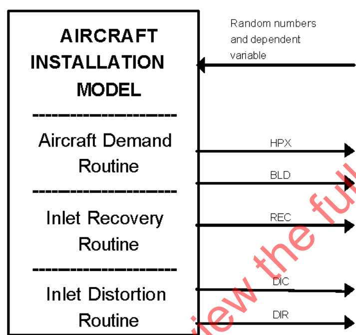


FIGURE 17 - AIRCRAFT INSTALLATION MODEL

5.3.1 Inlet Recovery Routine

The engine inlet corrected airflow is needed to determine distortion values. Thus, an initial guess of inlet pressure recovery is needed by the Engine Model in order to determine the engine corrected airflow. The pressure recovery value also allows the Engine Model to determine absolute pressure level. Inlet pressure recovery characteristics are normally based on scale model inlet test results and are modeled as a deterministic function of free-stream Mach number, angle of attack, angle of sideslip, and airflow. In the early design phases of a new vehicle, estimates based on past experience and CFD results (when available) can be used before scale model inlet test results become available.

5.3.2 Inlet Distortion Routine

Applicable inlet distortion descriptors are modeled as deterministic functions of free-stream Mach number, angle of attack, angle of sideslip, and airflow. The distortion descriptors must be supplied in a form that the engine model can use to determine the decrement in stability margin remaining for the applicable compression components (the fan and compressor in most cases). These distortion descriptors should be consistent with the methodology of ARP1420/AIR1419 and should be from the same database as the values used in the current practice stability audit. It is recommended that a default path (discussed in 7.1) be included in the statistical stability audit procedure to provide a comparison to current practice stability audit values (see Appendix B).

It is recommended that the stability assessment algorithms that determine the decrement in stability margin be included in the engine model and only the basic distortion descriptors are output from the aircraft model. Modifiers determined from appropriate distributions obtained from manufacturing statistics can account for inlet-to-inlet variability if the variation in pressure recovery and/or distortion is statistically significant.

5.3.3 Aircraft Off-takes Demand Routine

This routine is intended to determine the effect of engine bleed and horsepower off-take. Usually, horsepower off-take is specified as a fixed value for all flight conditions and bleed rates are a function of Mach, altitude, and power setting. Ideally, the bleed and horsepower off-take would be modeled as a function of altitude, Mach number, power setting, maneuver conditions, and possibly an appropriate distribution function. This distribution function could include dependencies on maneuver conditions and load shifting on multi-engine vehicles.

5.4 Engine Model

The Engine Model has as its core, the basic thermodynamic engine matching or cycle simulation. It also has an overlapping subroutine for performing the engine manufacturer's distortion handling methodology and the datum of clean inlet stability limit lines defined as a function of corrected airflow for all compression system components. The Engine Model calculates the stability margin available for handling the inlet distortion, bleed, and power off-take demands determined by the aircraft model. The effects of engine-to-engine variability and deterioration are also characterized within this model. The Engine Model also determines the stability margin consumed by the throttle transient selected by the Usage Model. It further includes the engine sensitivity to inlet distortion and thus the amount of stability margin consumed by the aircraft maneuver. A flow diagram of the engine model is shown in Figure 18. The core thermodynamic model may be a steady-state cycle deck or an appropriate dynamic model.

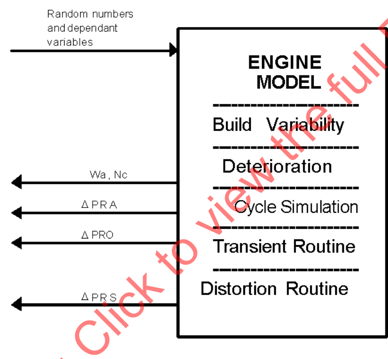


FIGURE 18 - ENGINE MODEL FLOW CHART

Many sources of data for the distributions used within the engine model are available. Development engines, which are normally highly instrumented, give a good account of component performance and fuel schedule variations. Production engines, although not as heavily instrumented, give a good account of the overall population variability. Control system components are also normally tracked in their acceptance test. These data can then be used in control system models to give overall fuel and variable geometry scheduling variations. Field service engines, from programs such as "lead the fleet," are another source of data for both engine variability and deterioration modeling. As discussed in 5.4.3, the level of component deterioration is normally based on accelerated mission endurance testing of the engine. When state-of-the-art engines are considered, estimates can be based on existing databases for engines of similar technology both in component and control system design.

5.4.1 Cycle Simulation

Steady-state and transient engine simulations are commonplace at gas turbine companies. These simulations model the entire engine on a component (fan, compressors, combustor, turbines, afterburner and nozzle) basis and generally include the control system. This type of simulation is a thermodynamic state model with maps representing each component. A typical component map format is depicted in Figure 19.

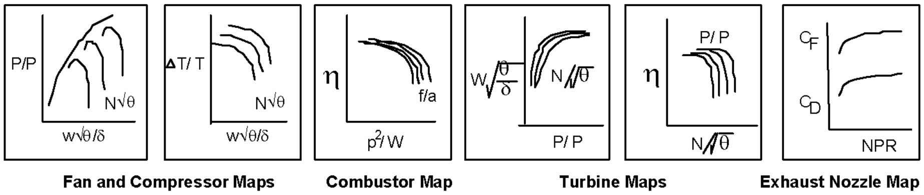


FIGURE 19 - TYPICAL COMPONENT MAPS FOR THERMODYNAMIC SIMULATION

Fundamental physical laws such as mass continuity, and work and energy balances are satisfied. A multitude of other parameters including efficiencies, speeds, pressure ratios, variable vane position, bleed flows, control feedback, and many others are provided. Adjustments for Reynolds number effects, tip clearances, off design vane position, etc. are commonly made to better represent component performance over a wide range of operating conditions. Modifiers to component maps are incorporated to handle input for engine-to-engine (build variability) and component deterioration. These will be discussed in detail in the following sections. Transient simulations account for inertia, heat transfer, and mass storage within each component. Volume dynamics within the engine can also be modeled if required.

These cycle simulations also require representations of atmospheric conditions, gas and fuel properties. Atmospheric temperature and pressure as discussed earlier may be represented by the 1976 U.S. Standard Atmospheric tables and the MIL-STD-210A (superseded by MIL-HDBK-310) hot and cold day tables. The cycle deck also requires input specification of the engine power requirements. The engine power setting requirements may be throttle position (or power level angle). Other inputs that come into play as off-take demands include bleed requirements for environmental cooling and external source extraction loads for generators and hydraulic systems.

Compression system stability lines are included with adjustments for known factors such as distortion, Reynolds number effects, tip clearances, heat transfer, inter-stage bleeds, off design stator vanes, spool interactions, and deterioration. The compression system operating lines are also affected by many variables and these are discussed in the following section. Stability margin available is then calculated for each compression component and provided as output for evaluation. It is not the intent of this document to go into more detail on the cycle deck. For more information see AS681.

5.4.2 Engine to Engine Variability

Engine-to-engine variability is associated with component performance variations due to manufacturing tolerances and later, deterioration as the fleet of engines ages. This thermodynamic variability causes variations in engine fan and compressor match, and thus stability margins. Engine-to-engine variations are accounted for by normally distributed variations about engine component performance parameters (maps), control schedule tolerances, and turbine flow areas as discussed in the preceding section. These may include the following items:

Fan Efficiency/Flow
Compressor Efficiency/Flow
High Pressure Turbine Efficiency
Low Pressure Turbine Efficiency
Exhaust Nozzle Flow Coefficient
Bypass Duct Pressure Loss
High Turbine Flow Function (Turbine Effective Flow Area)
Low Turbine Flow Function (Turbine Effective Flow Area)
Turbine Temperature (Control Limiting Function)
Fan Speed Schedule
Operating Line Schedule
Fan Inlet Guide Vane Schedule
Compressor Variable Stator Vane Schedule
Gas Generator Fuel Flow Schedule
Afterburner Fuel Flow Schedule

These performance parameters are varied by means of input modifiers to the component performance maps used in the cycle simulation. Values for these parameters are randomly selected, as discussed in 5.2.1, from their distributions and applied to the cycle simulation (see Figure 20). The standard deviation of the normal distributions for each of the component parameters may be determined from production engine data. The control and fuel schedule tolerances can be determined from engine or component bench testing or analytical estimates.

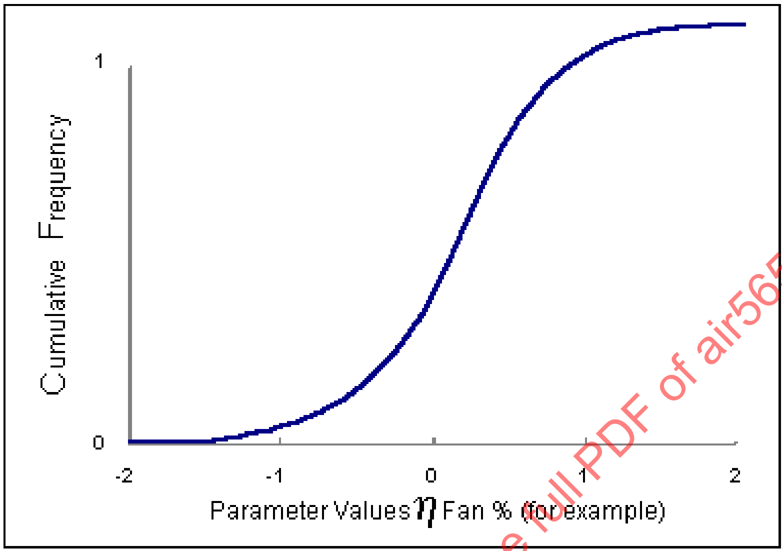


FIGURE 20 - BUILD VARIABILITY

5.4.3 Engine Deterioration

As the fleet ages, engine deterioration enters as a variable factor affecting engine matching. Engine deterioration is a thermodynamic shift in the cycle as the components lose efficiency due to tip and seal clearance increases that cause an increase in end wall losses and secondary flow leakage. This phenomenon occurs from the front to back of the engine with the high-pressure components (high-pressure compressor and turbine) having the greatest losses due to their more severe environments. The resulting thermodynamic shift in the cycle causes a rematch to occur on both the fan and compressor and, thereby, a change in available stability margins. The stability limit line pressure ratio of the compression components can also degrade with increases in clearance. The engine deterioration level is normally specified as a function of total accumulated cycles (TACs). The deteriorated engines are modeled by applying loss modifiers to the cycle decks, such as those described in ASME 97-GT-48. These modifiers are similar to build variability except the modifiers are always negative. Examples of variables to which these modifiers can be applied are:

Fan Efficiency/Flow
Compressor Efficiency/Flow
High Pressure Turbine Efficiency
Low Pressure Turbine Efficiency
Turbine Flow Functions
Turbine Temperature Sensing Bias with Time

The efficiency decrements are usually deterministic functions of engine mission cycles (TACs) (see Figure 21). The level of component deterioration is normally based on sea-level-static accelerated mission testing of the engine. This element will be discussed in more detail in the following section.

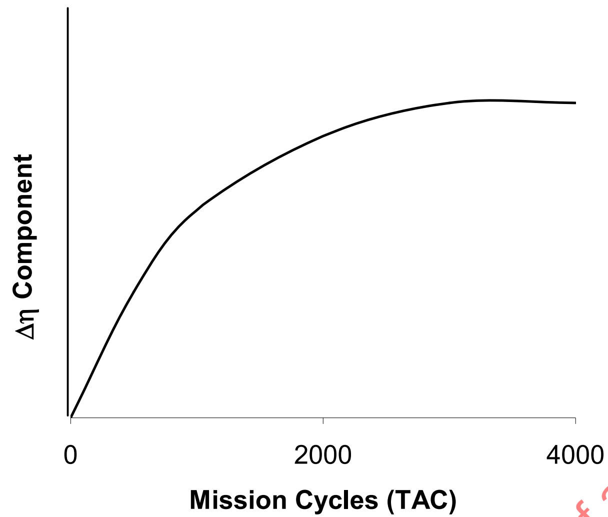


FIGURE 21 - COMPONENT DETERIORATION

5.4.3.1 Fleet Engine Deterioration

As in the previous section on variability, each component's thermodynamic characteristics (efficiency and flow capacity) starts in a physical model with an initial build variability. Initial build variabilities start with fairly tight distributions (sigma value is small), but as an engine enters service, deterioration begins almost from the very first flight or usage. Transients and "g" loading will begin to increase component tip clearances, thus lowering efficiency and stability limit lines. High turbine temperatures initiate the process of erosion in the turbines, again opening clearances and lowering efficiencies. Foreign Object Damage (FOD) also is a source of component degradation along with environmental conditions in certain theater specific operation such as sand exposure. All of these items contribute to deterioration of the components. As this deterioration process proceeds, the engine will rematch itself in the compressor and fan to higher operating pressure ratios and thus lowering available stability margin. A graphical depiction showing the manner in which the initial component efficiency and build variability degrade with time is presented in Figure 22.

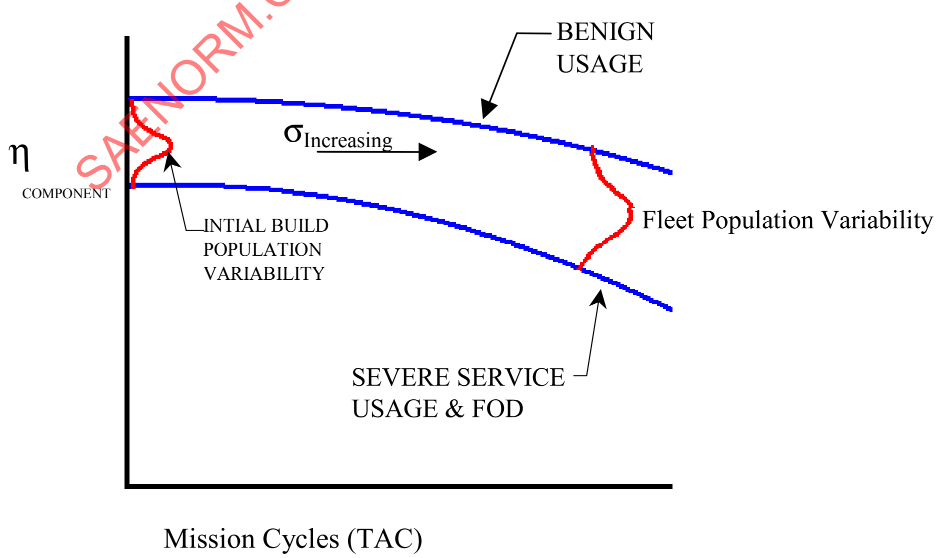


FIGURE 22 - EFFECT OF FLEET ENGINE DETERIORATION ON COMPONENT EFFICIENCY

Military aircraft engine usage is tracked in terms of TACs as mentioned above. Other gas turbine applications may have their own unique tracking parameter for usage accumulation with time but the deterioration process with time discussed above would be generally the same.

The effect of deterioration on available stability margin is shown in Figure 23. The stability limit line is also degraded by tip clearance increases, foreign object damage (FOD), and erosion, while the operating line shifts to a higher pressure ratio.

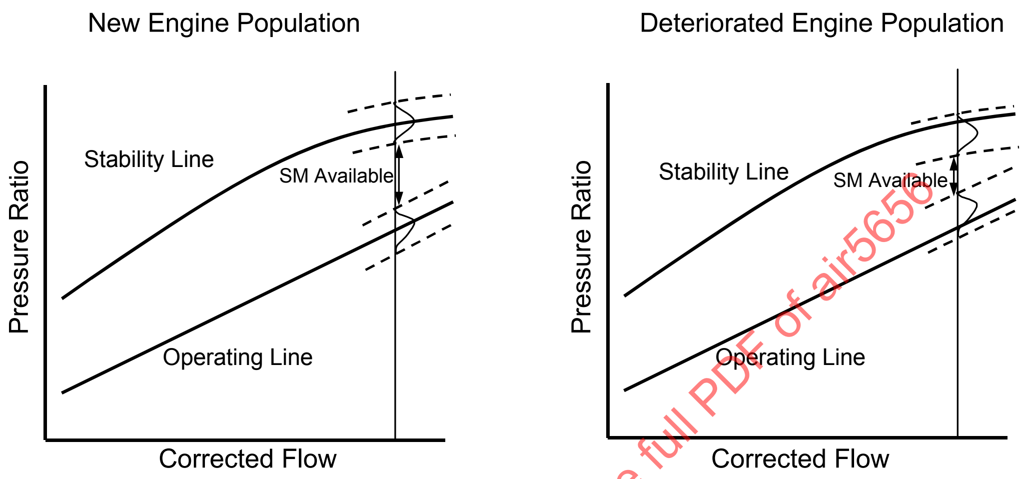


FIGURE 23 - DISTRIBUTIONS FOR NEW AND DETERIORATED ENGINE POPULATION

This process of component deterioration with time is physically modeled in the cycle deck and therefore, one can trace the impact of deterioration on available stability with usage.

5.4.3.2 Population Distributions

In doing risk management studies with a fleet or population of engines, it is also important to note and model (with an appropriate distribution) that all engines do not enter service at the same time. There will be a distribution of age in the fleet. This is shown schematically in Figure 24. It should be noted that this distribution represents a snapshot of the fleet age at the time of interest and would need to be updated as the fleet ages and demographics change.

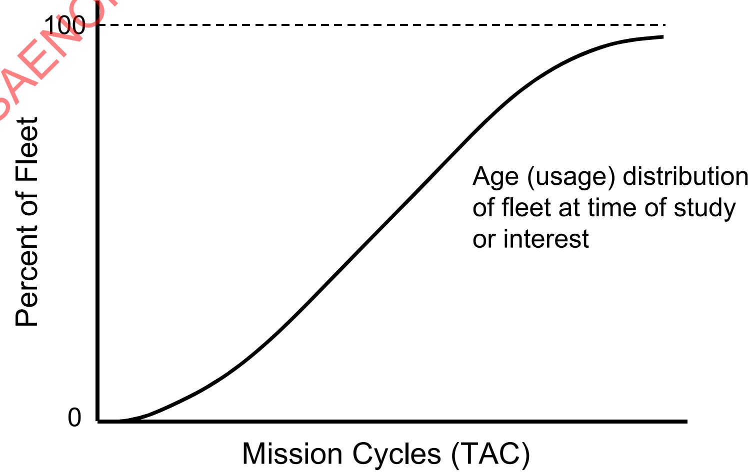


FIGURE 24 - AGE DISTRIBUTION OF FLEET AT TIME OF STUDY OR INTEREST

5.4.3.3 Impact of Overhaul

Further consideration should also be given to the fact that the fleet population of engines will at some point become a mix of new engines, and overhauled engines. Overhauled engines have been refurbished and regained some degree of their initial performance and have then their own unique component efficiency distributions and lapses with usage.

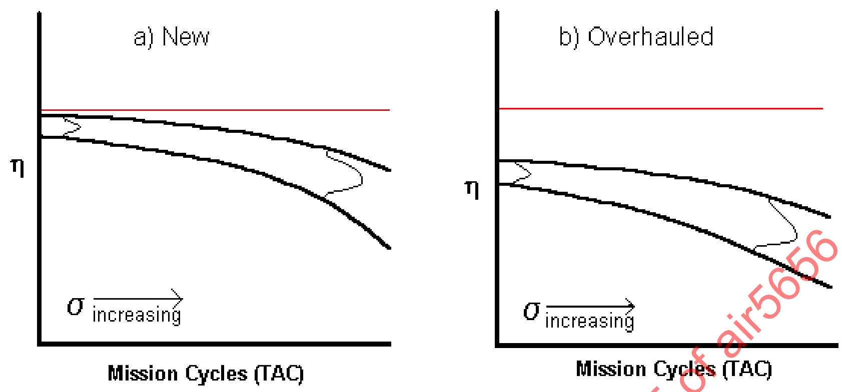


FIGURE 25 - DETERIORATION LAPSE RATE FOR NEW AND OVERHAULED POPULATIONS

The distribution of new engine performance (as shown in Figure 22) is shown in Figure 25a, while the degraded performance characteristics of an overhauled population of engines is shown in Figure 25b. Engines are generally not refurbished to new performance levels, primarily due to cost constraints. Furthermore, the distribution coming out of overhaul is also wider than the distribution of new engines ($\sigma_{OH} > \sigma_{NEW}$), because the degree of overhaul varies widely depending on the need of each engine as it passes through overhaul. Engines have an initial usage (time) distribution and age distribution, however once the fleet starts passing through overhaul, another usage distribution and age distribution that models the overall fleet of engines will be used.

One final point should be made when considering fleet performance deterioration. Generally, engines are bought at specification performance levels with minimum values set. This may lead to truncated distributions in the initial population for zero time. This is also true of engines out of overhaul because they generally must pass certain quality filters at acceptance. This process is depicted in Figure 26. This would lead to a truncated distribution in the component performance distribution. In general, the high-pressure spool has the most significant deteriorated components and therefore, the initial distributions for the high-pressure compressor and turbine would start out with truncated distributions.

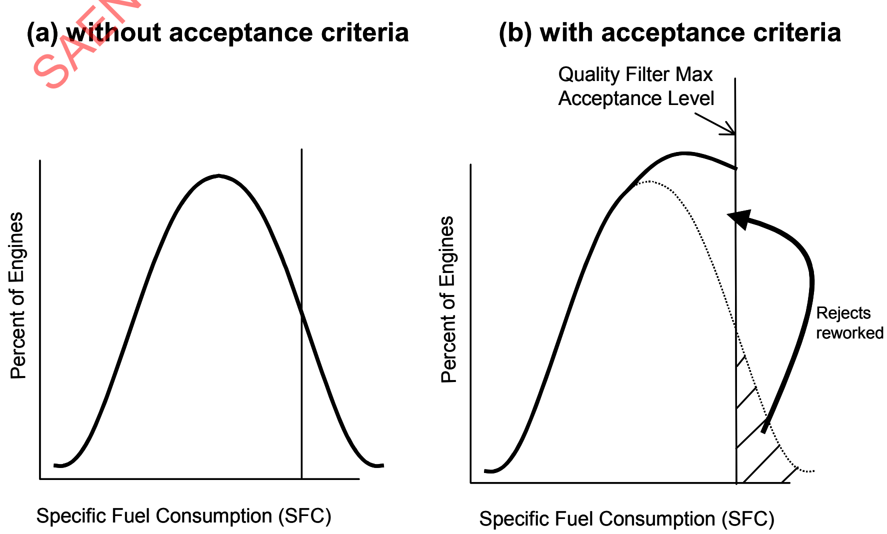


FIGURE 26 - SFC DISTRIBUTION WITH AND WITHOUT ACCEPTANCE CRITERIA

5.4.4 Transient Model

The Transient Model captures one of the significant destabilizing variables in the stability audit as transient operation can consume a large amount of both fan and compressor stability margin (see Transactions of ASME). A typical compressor operating line excursion during engine acceleration (Type 2 transient) is shown schematically in Figure 27. Starting at the steady-state operating point, an increase in fuel flow causes the compressor to operate at a higher pressure ratio with little increase in rotational speed. The engine then starts to accelerate and the operating point moves to the right along the transient operating line. The amount of stability margin consumed during such a transient is determined by the acceleration fuel schedules, which are set by the amount of unbalanced torque required to meet the transient time requirements. The more aggressive the acceleration, the more stability margin consumed. The inherent pinch points that occur in this type transient are shown. One is a low-power, off-idle pinch point, which occurs early in the transient at low power. The latter occurs at near maximum airflow where distortion also becomes a significant contributor to stability margin consumption.

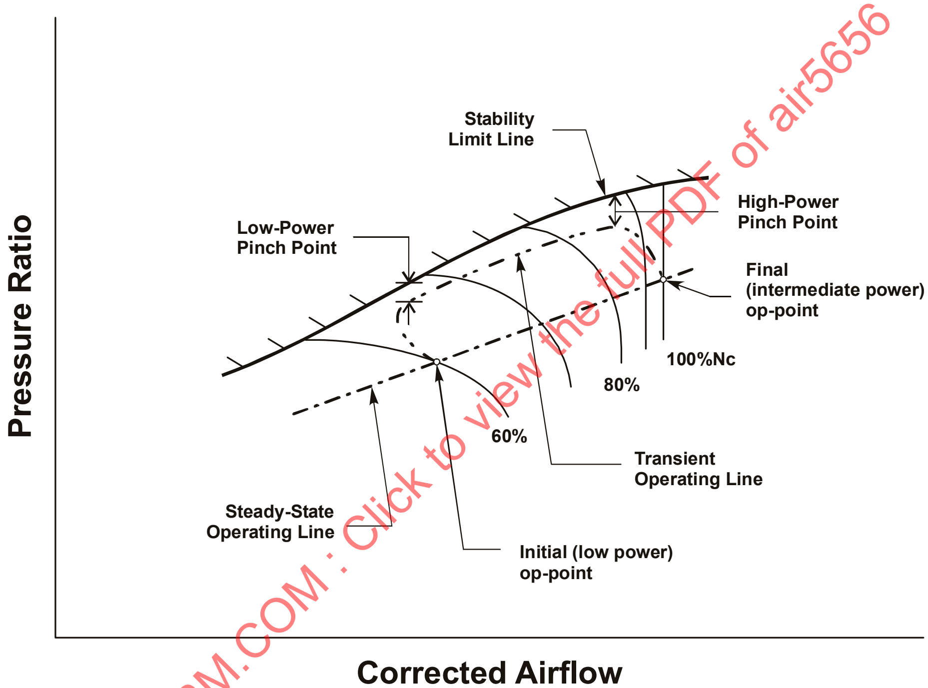


FIGURE 27 - TYPICAL GAS GENERATOR ACCELERATION (TYPE 2 TRANSIENT)

The stability margin consumption for each throttle transient type may be determined from engine test. If a steady-state cycle deck is used to provide the clean stability margin, the margin consumed by the throttle transient must be supplied by this separate routine.

Each engine model will have a unique transient operating line for a given throttle transient. However, other factors will introduce transient operating line variability. The most significant factor is the type of fuel scheduling used in the engine control system. It can be modeled as normally distributed about a mean value as illustrated in Figure 28. In open-loop fuel scheduling, based on cams in hydro-mechanical fuel controls, this variability can be significant (3 to 4%). In closed-looped control systems, which are common in advanced full authority digital fuel controls, this variability is reduced to 1 to 2%.

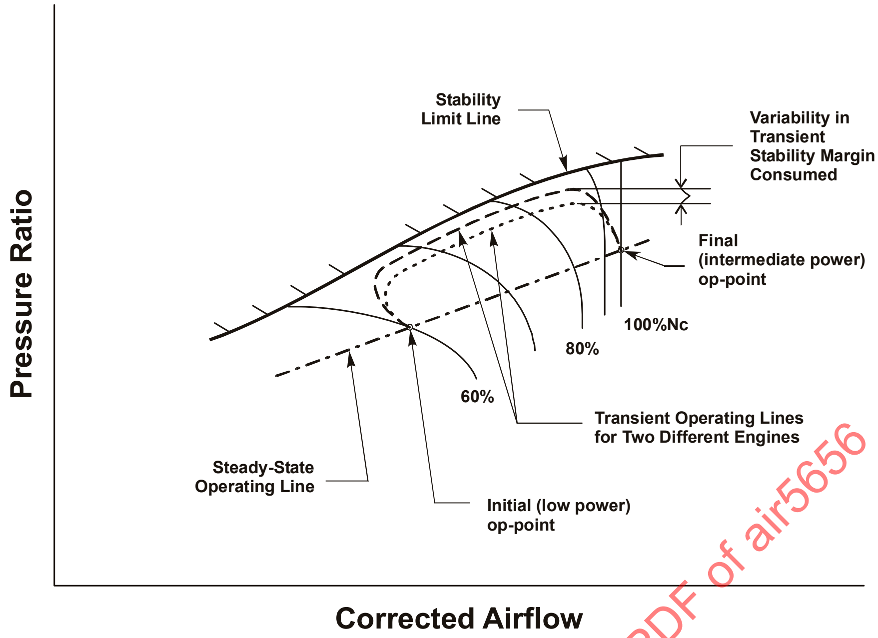


FIGURE 28 - FUEL SCHEDULING VARIABILITY

The implementation of this variability in the engine model is handled by distributions for specific engine corrected airflows as shown schematically in Figure 29. The number of airflow lines would be dependent on the fidelity needed to capture the distribution across the entire airflow range from idle to intermediate power. A third variability on this element can be added which is an altitude bias. It recognizes that the sensitivity to fuel scheduling errors is greater as altitude increases because the absolute error becomes a larger percentage of the average fuel flow rate required to perform the acceleration. This can be added as a deterministic modifier to the output of Figure 30, which depicts a baseline database (usually sea level static).

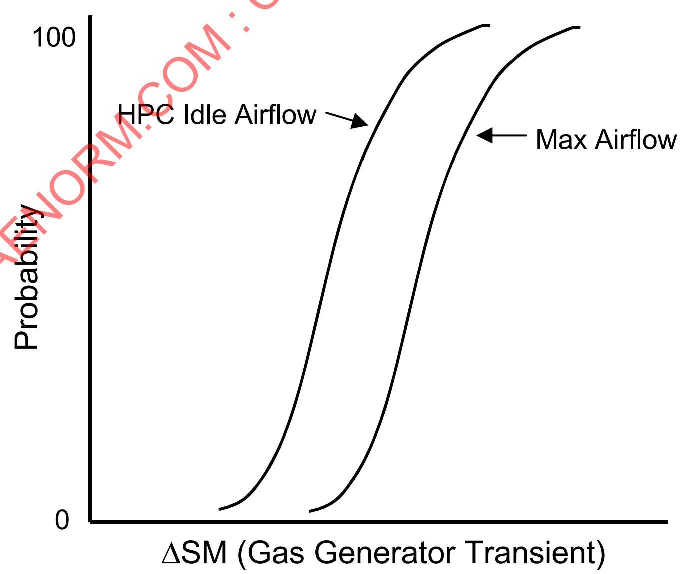


FIGURE 29 - TRANSIENT STABILITY MARGIN MODEL

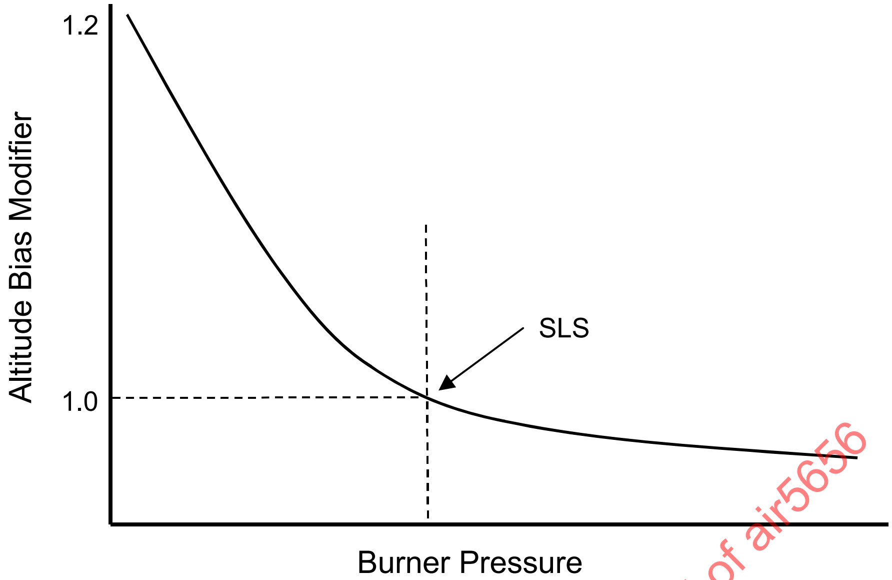


FIGURE 30 - ALTITUDE BIAS (NOTIONAL)

Augmentor transients (a Type 1 transient from the throttle usage model) are modeled in much the same manner as the gas generator transient with the exception that there is one less variable because all A/B transients occur at or near intermediate airflow. Therefore, modeling for the augmented transient is handled as in Figure 31.

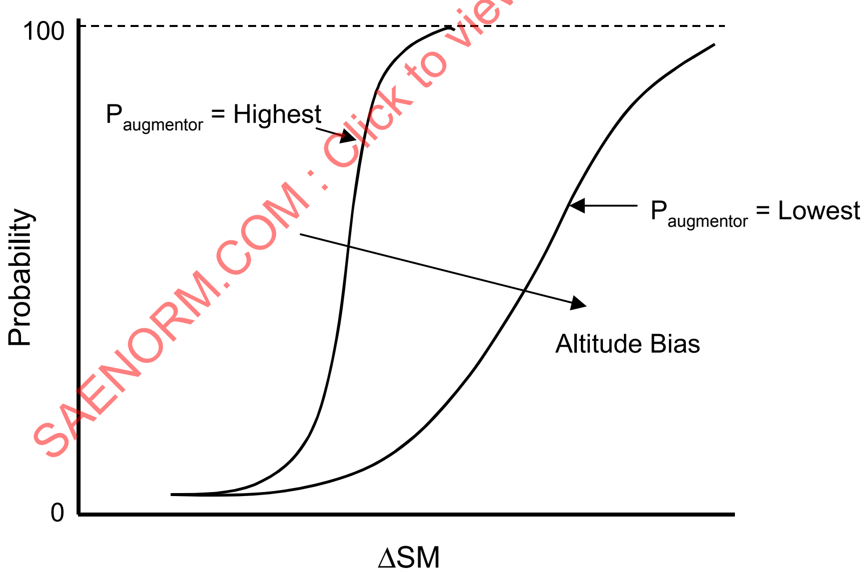


FIGURE 31 - AUGMENTOR PRESSURE SPIKE

If a higher fidelity transient engine model is used to thermodynamically represent the engine, this modeling element is not needed as the physics of this element is captured in the model. Variability maybe handled by adding fuel-scheduling distributions.

5.4.5 Inlet Distortion Routine

As mentioned (5.3.2), the inlet distortion routine is consistent with the engine manufacturers' distortion methodology for the engine. It performs the same deterministic analyses as the current practice stability audit, with the exception that the values used for the variable elements come from the distribution embedded with each supporting model. A flow diagram of this element alone is shown in Figure 32.

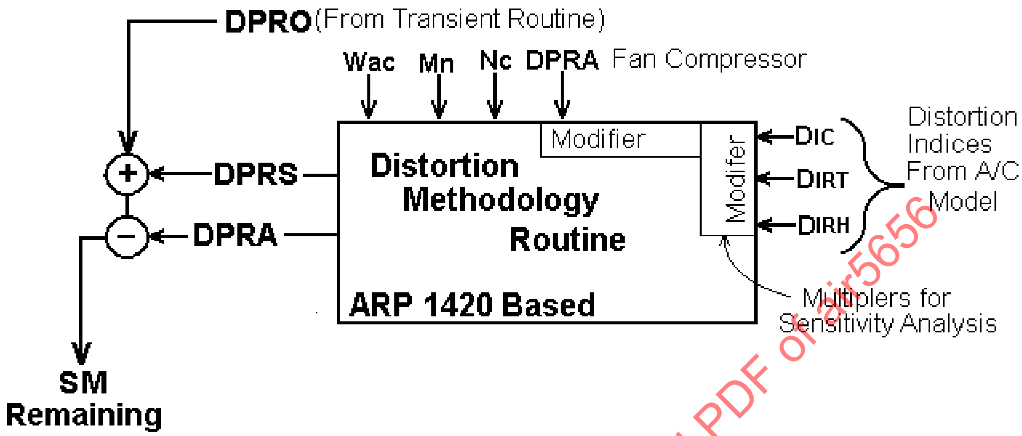


FIGURE 32 - DISTORTION METHODOLOGY FLOW DIAGRAM

Correlations relating loss of fan and compressor stability margin to the inlet distortion descriptors and engine airflow are developed following ARP1420/AIR1419 practice. Both rig test and full-scale engine test data, obtained with classical and complex inlet distortion pattern screens, are used to develop these correlations. Typically distortion methodologies are defined to predict loss of stability pressure ratio to within ±2%. This uncertainty should be included in the distortion accounting. Provisions can be made for the user to apply multipliers to the circumferential, hub radial, and tip radial sensitivities or their distributions. These can be applied to clean (undistorted) fan and compressor stability margins to study impacts of potential compression system redesign effects on stall rate and/or inlet modification, as they would change distortion stability margin consumption.

Although the model depicted in Figure 32 only shows total pressure (spatial) distortion, it can equally be extended to handle other types of distortion such as temperature and planar waves. Temperature and planar distortion are discussed in detail in ARD50015 and AIR5866 respectively.

The final calculation that remains is the summing of the destabilizing elements in the audit process, the engine transient (DPRO) and the distortion induced stability pressure ratio loss (DPRS), and subtracting the result from the delta pressure ratio available (DPRA) from the engine cycle. This summing routine is shown in Figure 33.

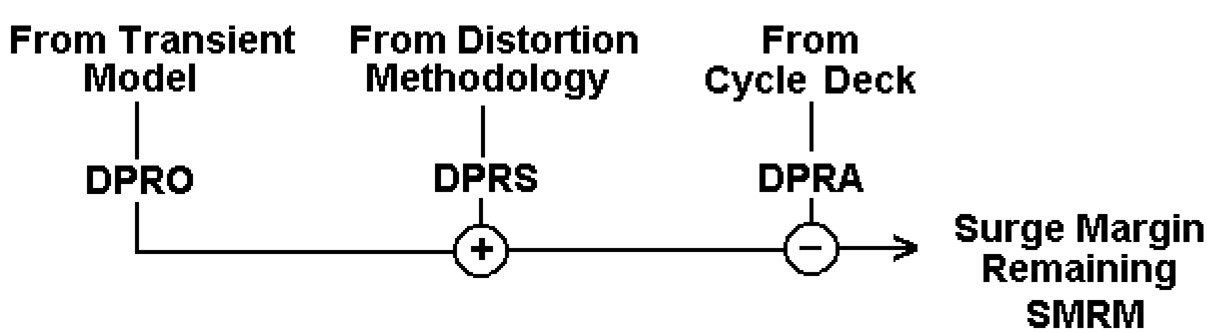


FIGURE 33 - STABILITY MARGIN REMAINING SUMMING ROUTINE

The stability margin remaining is stored to a file with other relevant variables for the case to be interrogated later. The simulation is rerun for a large number of cases, each providing relevant stability data. Determining the number of cases is discussed in 6.4.1.

6. APPLICATION OF STATISTICAL STABILITY ASSESSMENT

Statistical Stability Assessments have been applied during a number of airborne weapon system programs. In this section, examples are presented from actual simulations that have been developed and applied to provide insight to the risk of in-service stall events. The output from the SSA simulations was used in risk assessments enabling stall rates to be managed to an acceptable level.

6.1 Simulation Development

The statistical stability assessment simulation, used in the following sections, is representative of a fighter aircraft weapon system with an afterburning turbofan engine, and will be from here on referred to as a “notional aircraft system.” It is described in more detail in 6.3. The simulation was configured similar to that shown in Section 4, but the executive routine was embedded in the engine model as a subroutine. A depiction of the simulation architecture is shown in Figure 34. All of the models were constructed consistent with the methodologies outlined in Section 5.

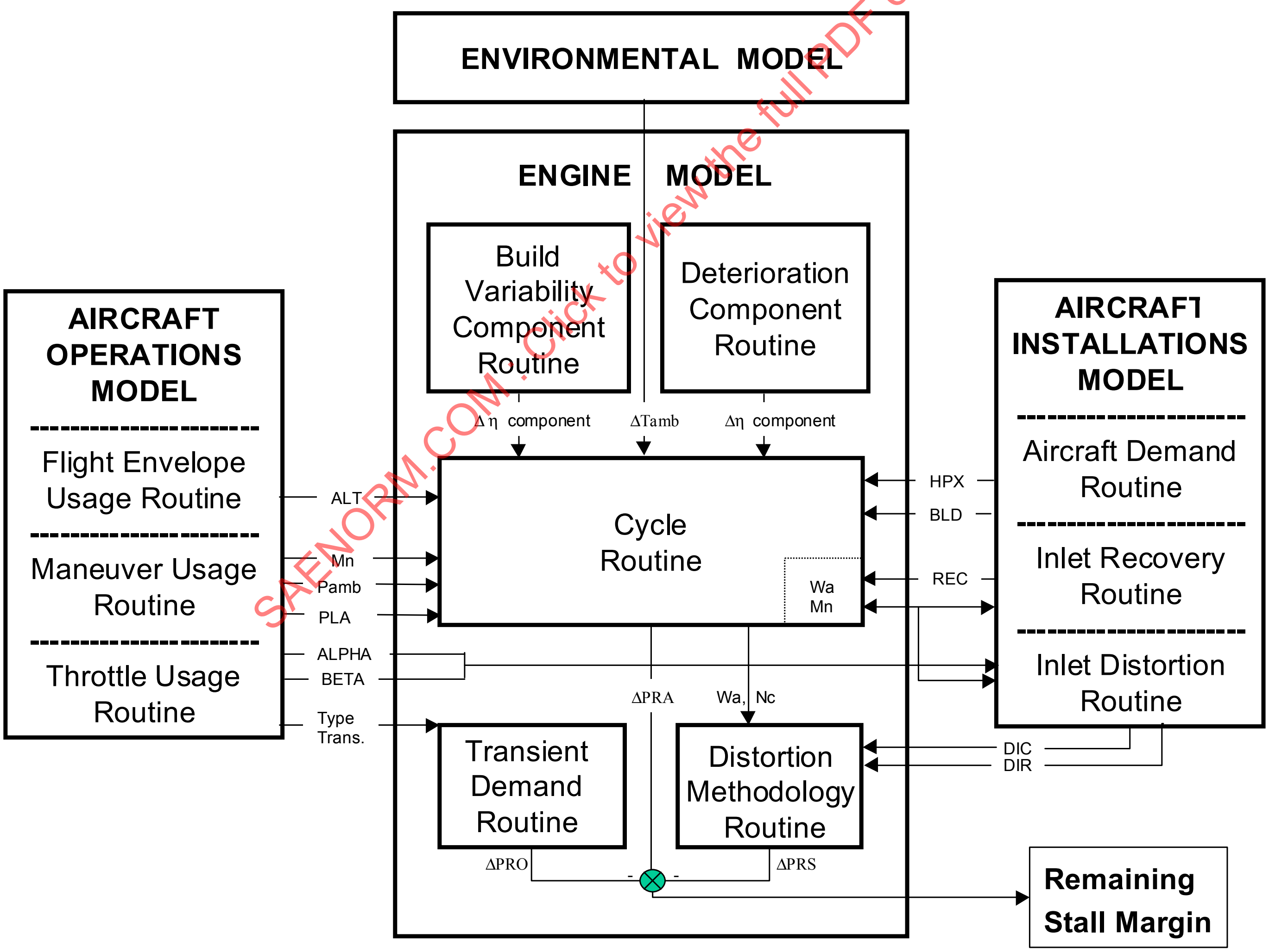


FIGURE 34 - CASE STUDY SYSTEM ARCHITECTURE

The environmental model was simplified and only considered the United States Hot, Cold, and Standard Atmospheres as contained in MIL-STD-210A.

The usage model was compiled from flight recorder data traces, as depicted in Figure 35, into the form shown in Figure 8.

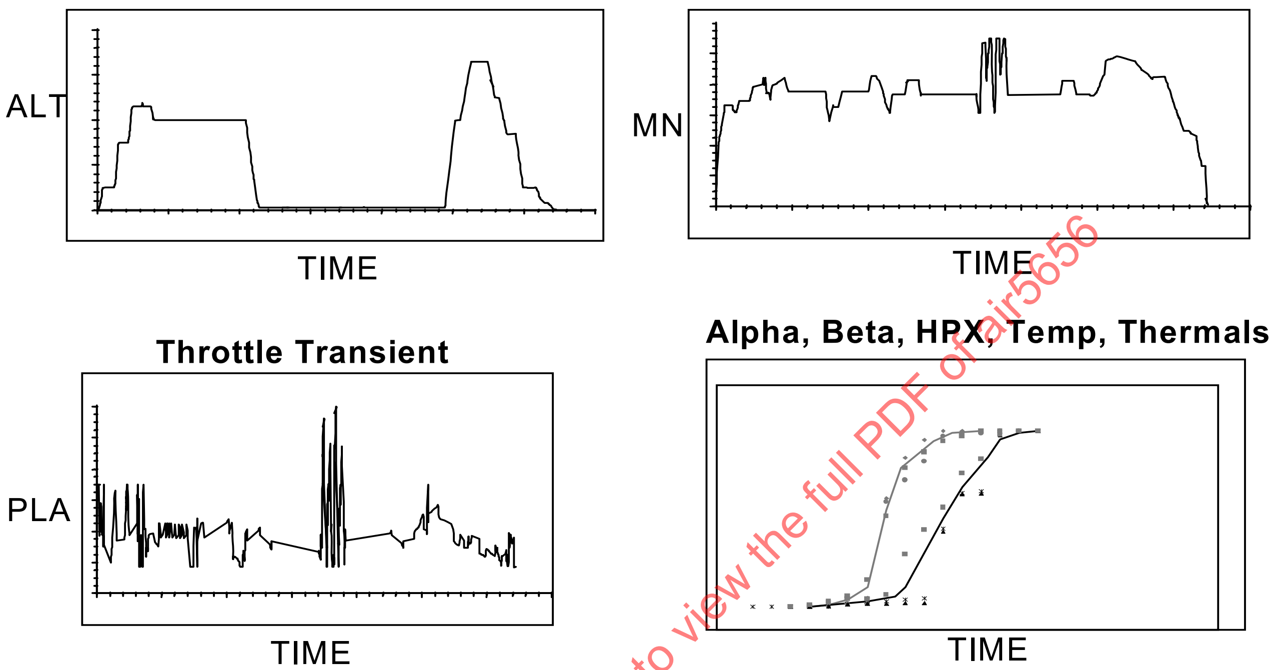


FIGURE 35 - FLIGHT DATA FOR CHARACTERIZING MISSION USAGE

The aircraft model contained the inlet distortion characteristics, pressure recovery, bleed, and power off-take demands imposed on the engine. It was developed as described in 5.3. The inlet characteristics were based on inlet scale model testing with recovery and distortion indices defined as a function of Mach number, angle of attack, angle of side slip, and corrected air flow. The distortion indices were derived for the specific engine consistent with ARP1420 methodology.

The engine model was consistent with that outlined in 5.4. The engine cycle routine was the core of the model and had the capability to accept all of the component modifiers and engine build variants described in 5.4.2. A fleet engine deterioration subroutine was also provided that enabled the effect of the fleet's age distribution on stall rate to be determined. The clean stability limit lines were built into the cycle deck fan and compressor maps. The output from the cycle routine was the stability margin available to tolerate engine transients and inlet distortion.

A full transient simulation of the engine was not used in this application. The measured transient characteristics of the engine were modeled in the manner described in 5.4.4.

Also included in the engine model was a distortion routine as described in 5.4.5. The inputs and outputs were as shown in Figure 32. The engine model, therefore, had the capability of accepting all supporting model inputs for a selected case as described in the overall system architecture, Figure 34.

Output from the simulation was the stability margin available, the stability margin required for the destabilizing factors, and the remaining stability margin. If stability margin required exceeded that available, a stall event was predicted.

6.2 Simulation Validation

Before exercising the simulation, it was necessary to validate the output. Checks were made of the individual model output distributions against those specified and the sample size was optimized as described in 6.2.2.

6.2.1 Individual Model Output

The initial step of the validation process was to determine the degree of replication of the original input data. Validation of the flight-envelope usage routine was discussed in detail in 5.2.1. The usage routine was based on flight recorder data and was shown in Figure 8. Adequate samples were run to ensure the output distribution matched the input. The flight-envelope usage data from a run of the simulation is shown in Figure 36. The degree of replication of the model within a given altitude and Mach number block was shown in Figure 11 and Figure 13.

The aircraft model also contained a maneuver usage envelope consistent with that shown in Figure 14. Angle of attack and sideslip were modeled as a function of Mach number and are truncated at the limits shown. The AOA and AOSS truncated distributions for the 3000 sample cases (see 6.2.2) are shown in Figure 37.

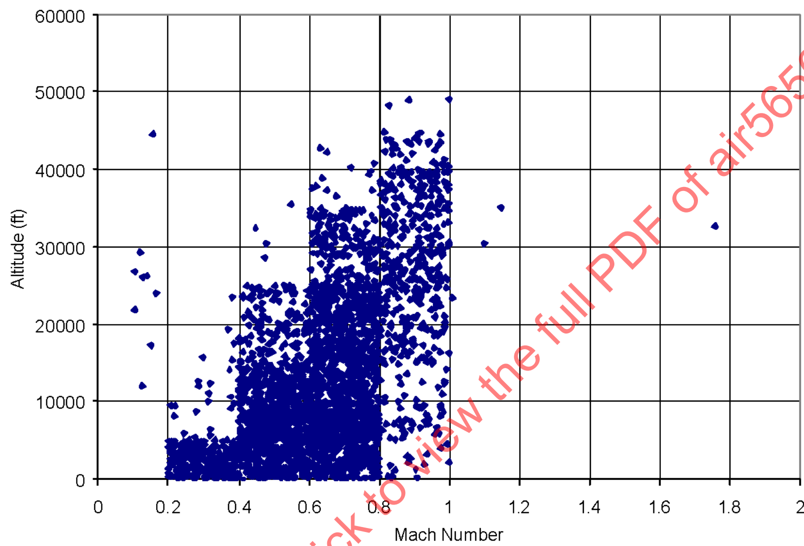


FIGURE 36 - PREDICTED FLIGHT ENVELOPE USAGE

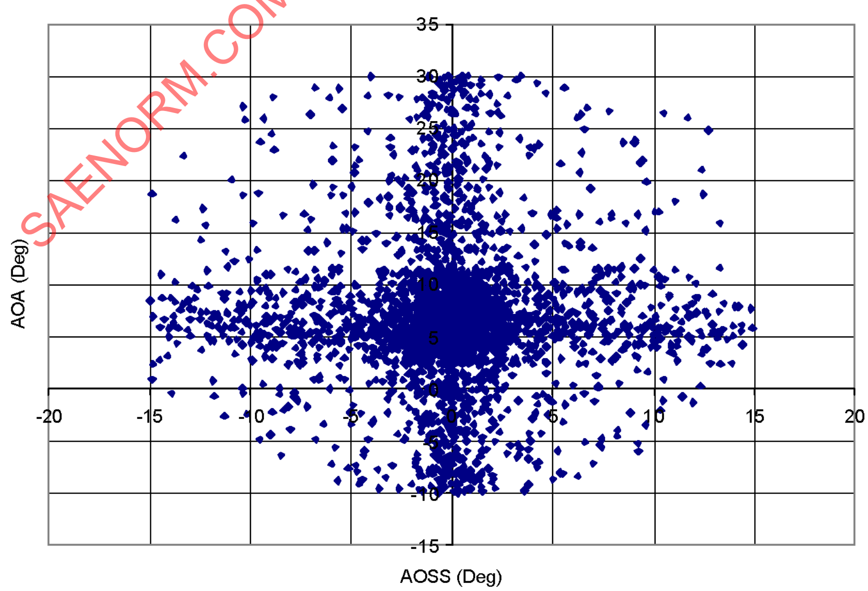


FIGURE 37 - PREDICTED ANGLES OF ATTACK AND SIDESLIP

The output from the Usage Model was used in the Aircraft Model to determine the inlet recovery and distortion. The circumferential and radial distortion indices output from the Aircraft Model are shown in Figure 38. For most cases, the distortion levels are modest. But a check was made to ensure that the extreme, or worst case, events had been captured.

A histogram of the throttle transient types obtained is displayed in Figure 39. Again this was checked to ensure it reflected the specified input distributions. Afterburner cancellation transients (type 5b) were not considered in this application.

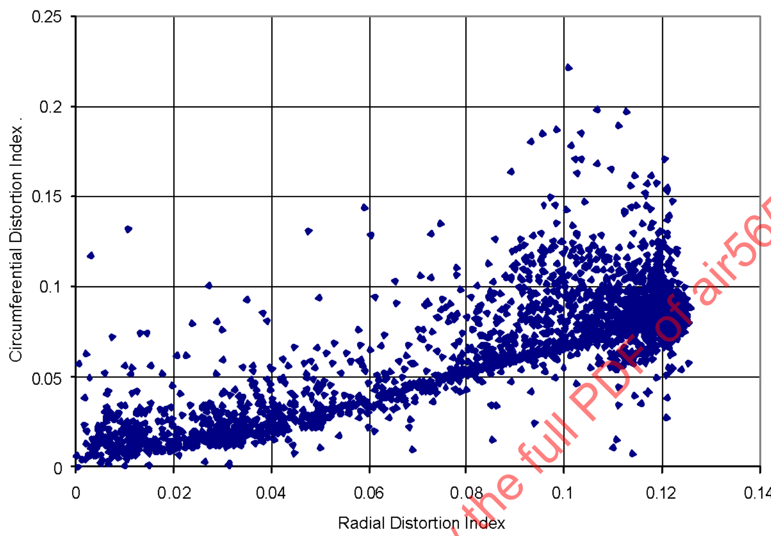
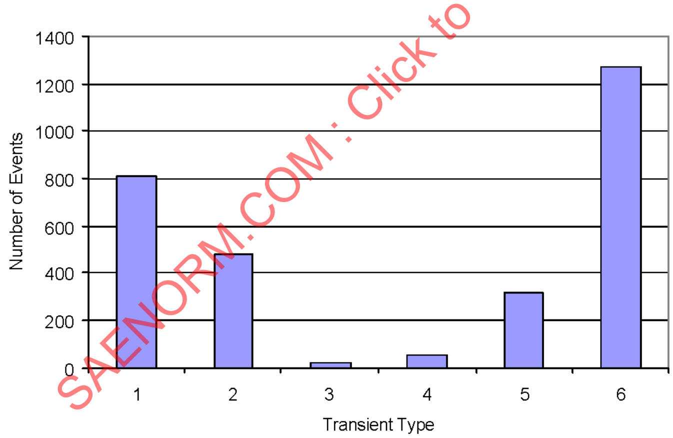


FIGURE 38 - PREDICTED DISTORTION INDICES



Transient Type	Description
1	High power accel
2	Low power accel
3	Bode
4	Decel
5	A/B/ Light
6	Fixed-throttle

FIGURE 39 - PREDICTED TRANSIENT TYPES (3000 SAMPLES)

The output distributions for all of the other parameters were interrogated and validated in a similar fashion to that shown above for the Usage Model.

6.2.2 Determination of the Sample Size

Before execution of the SSA simulation, it is necessary to determine an adequate sample size for good repeatability. Fan stall margin output from the simulation was used to check for the necessary sample size because it was the most limiting of the two compression system components. Sample sizes of 100, 500, 1500, 3000 and 9000 were run. For each sample size, a statistically significant number of cases were run to calculate the mean and standard deviation. The difference between each case was the seed value used (see Appendix A.1) to start the sequence of random number selections. For sample size 100, 40 cases were run; for sample size 500, 30 cases were run; for the remaining sample sizes, 20 cases were run. At each sample size, a data set was then generated and the mean and standard deviations calculated. Adequate repeatability was defined as the $\pm 2\sigma$ variation (which represents the 95% confidence line) for the sample size data set being within ±2% of the mean values. The tabular results are shown in Figure 40 and a graphical display is shown in Figure 41. For the data set of the 3000 sample size, the above criteria were met and this level of repeatability was considered adequate.

Sample Size	+/- 2σ	+/- 2% case mean
100	15.20	13.99
	12.24	13.44
500	14.33	13.98
	13.08	13.43
1500	14.05	14.04
	13.46	13.48
3000	14.00	14.05
	13.55	13.49
9000	13.95	14.07
	13.62	13.51

FIGURE 40 - INITIAL SAMPLE SIZE RESULTS

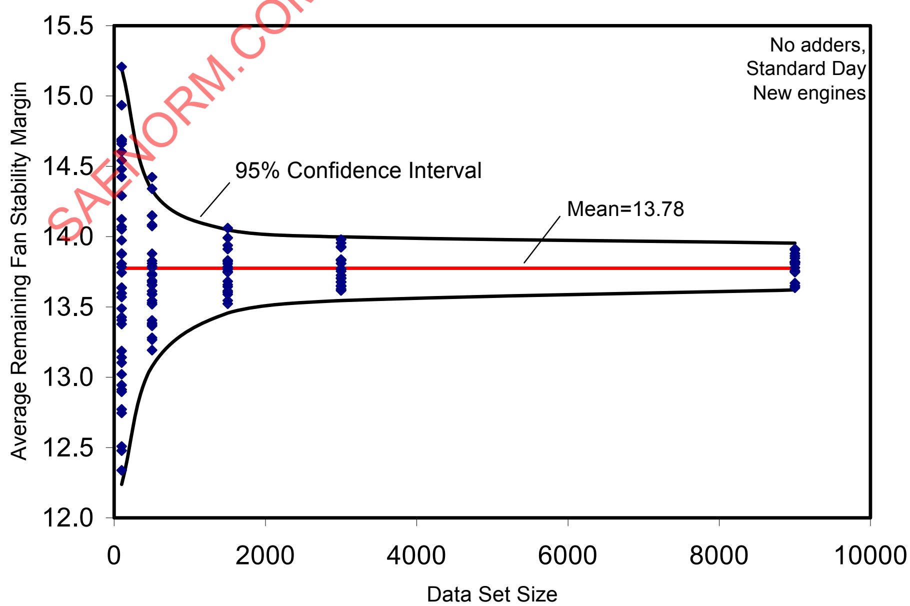


FIGURE 41 - SAMPLE SIZE VERIFICATION PLOT (IN SERVICE ENGINE - NO MODIFIERS)

This sample size of 3000 was then used for the remaining example for detailed analysis and is consistent with the data sets previously shown in Figures 36 to 39.

6.2.3 Comparison of Simulation With Service Experience

The resulting simulation predicted that no stalls would occur given the usage, installation, and engine models used. The stability margin remaining over the life of the fleet was positive. This was consistent with the field experience for the simulated weapon system in which stalls do not occur within the modeled distributions of the destabilizing variables. The model had an average remaining fan stability margin of 13.8% (Figure 41).

6.3 Notional Aircraft System

To make the simulation representative of a marginal system, modifiers were placed on both the fan and compressor clean stability limit lines and also on the inlet distortion parameters until a marginal, non-zero, stall rate was encountered. With these modifiers the average remaining fan stability margin was lowered to a little over 9% (Figure 42) and the resulting nonzero stall probability was approximately 0.5% stalls per sample (Figure 43). This enabled the simulation to be used to explore the effect that changing parameter distributions had on stall rate.

For the notional aircraft system (with modifiers added) the sample size determination was repeated and the 3000 case sample size was again judged acceptable for the remaining examples which will be discussed in the following section (see Figure 42 and Figure 43).

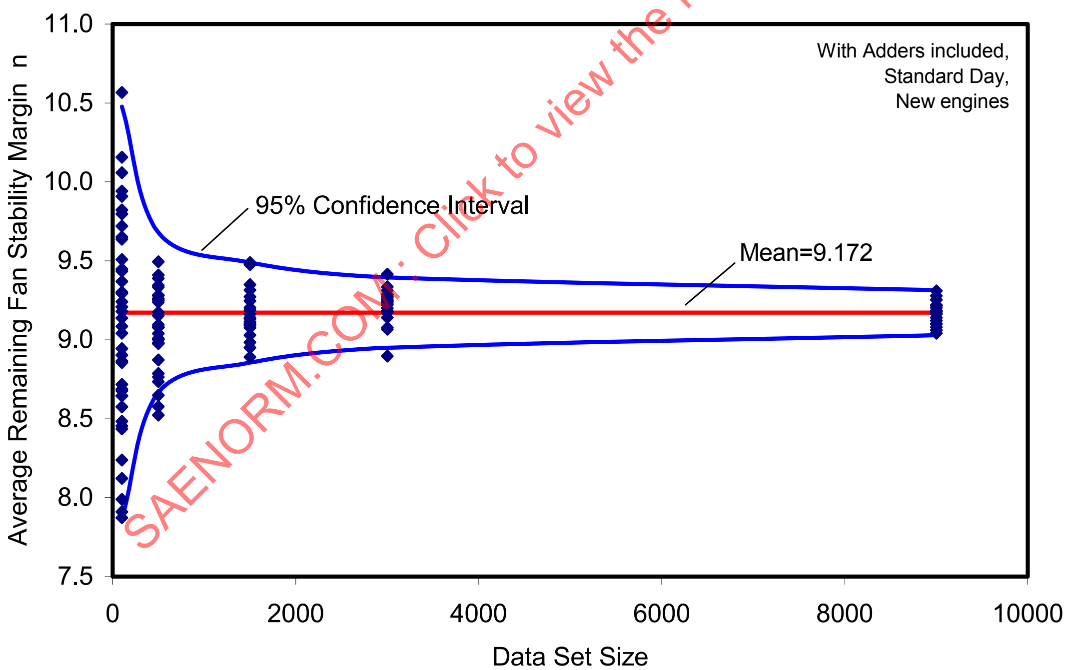


FIGURE 42 - AVERAGE REMAINING FAN STABILITY MARGIN (NOTIONAL A/C SYSTEM)

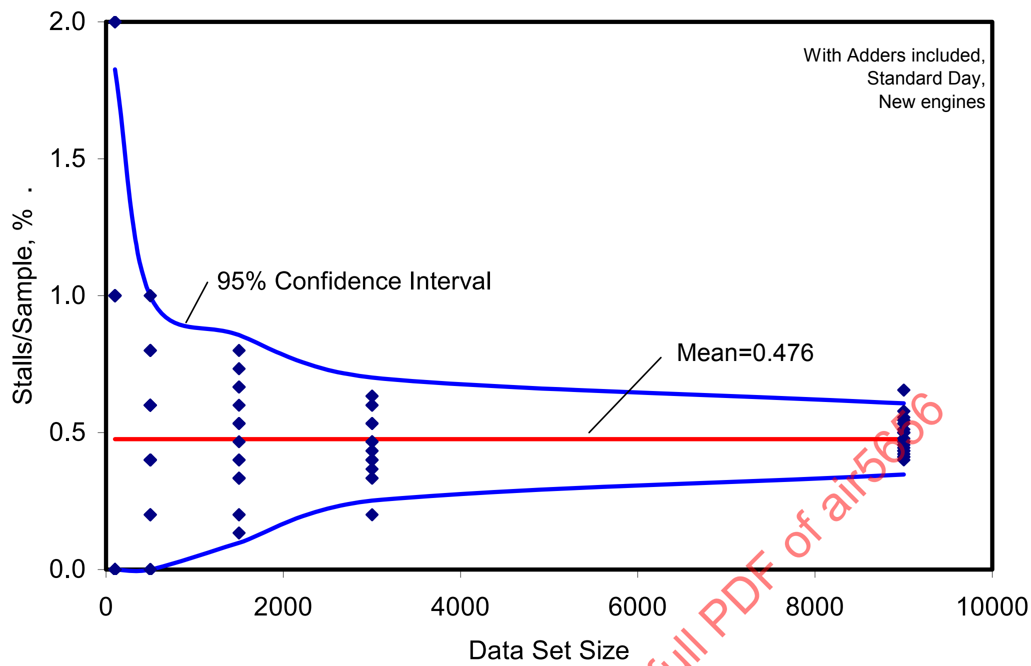


FIGURE 43 - STALLS/SAMPLE PERCENTAGE (NOTIONAL A/C SYSTEM)

6.3.1 Stability Prediction for the Notional Aircraft System

The following results show the manner in which the simulation can be used to understand what type of stalls are being encountered and where modifications would be needed to correct the unacceptable stall rate in fielded operations.

6.3.2 Fan Stall Events

The predicted values of remaining fan stability margin are shown in Figure 44 and a histogram of these predictions is shown in Figure 45. The long "tail" extending to the right in Figure 45 consists entirely of data points with relatively low fan airflow. The tail of a distribution refers to the low probability events with values, in this case remaining fan stability margin, that is significantly greater or less than the mean (Appendix D.5 and D.8). The fan-pressure-ratio match at low power results in high remaining stability margins, while the fan is matched up in pressure ratio at high power for performance. From the 3000 cases approximately 25 stall events were predicted.

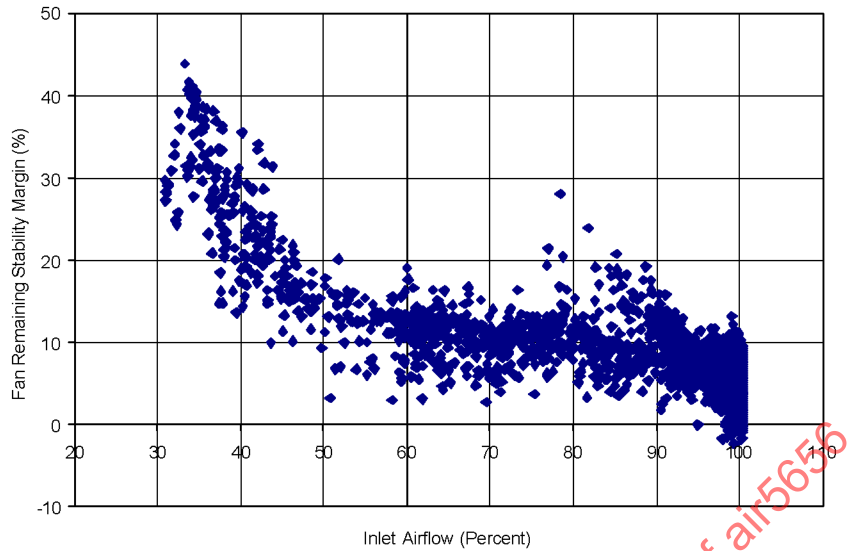


FIGURE 44 - PREDICTED FAN STABILITY MARGIN REMAINING

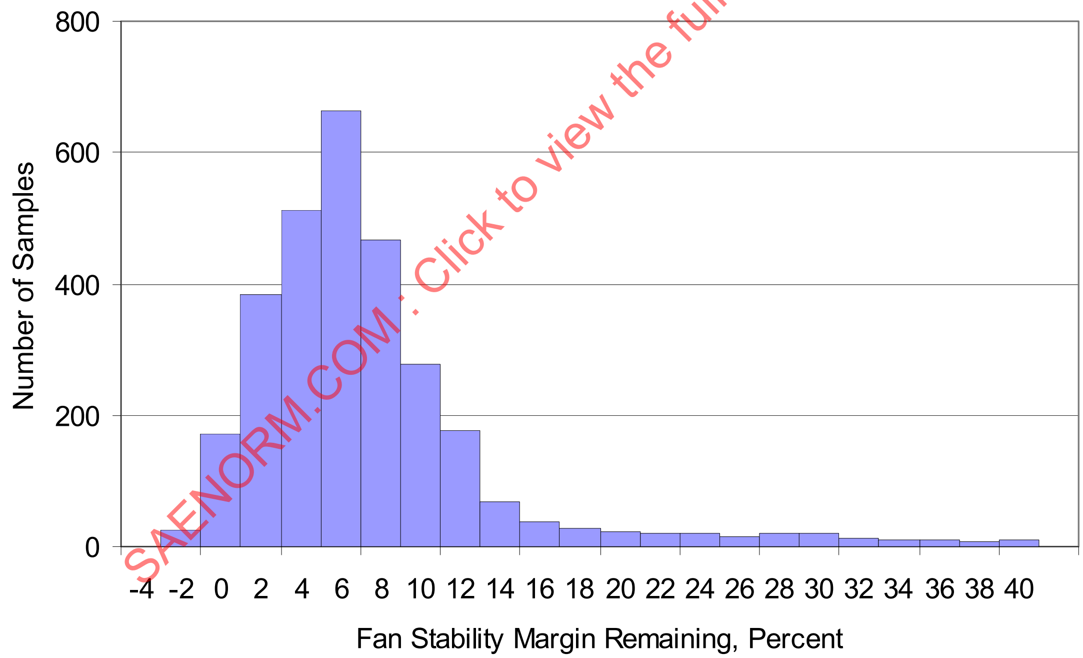


FIGURE 45 - PREDICTED FAN STABILITY MARGIN REMAINING HISTOGRAM

6.3.3 Compressor Stall Events

The predicted values of compressor stability margin remaining are shown in Figure 46 and a histogram of these predictions is shown in Figure 47. Overall, six compressor stall events were predicted. From the highly skewed distribution in Figure 47, it may be concluded that the compressor has excessive stability margin across the full power range. Two types of stall events are illustrated in Figure 46. The first type is off-idle stall driven entirely by fuel acceleration schedules and variability. The other stalls occurred at high power where inlet distortion and acceleration fuel scheduling were the major destabilizing factors. This information gives insight into what corrective action to take if the stability of the high-pressure compressor becomes unacceptable.

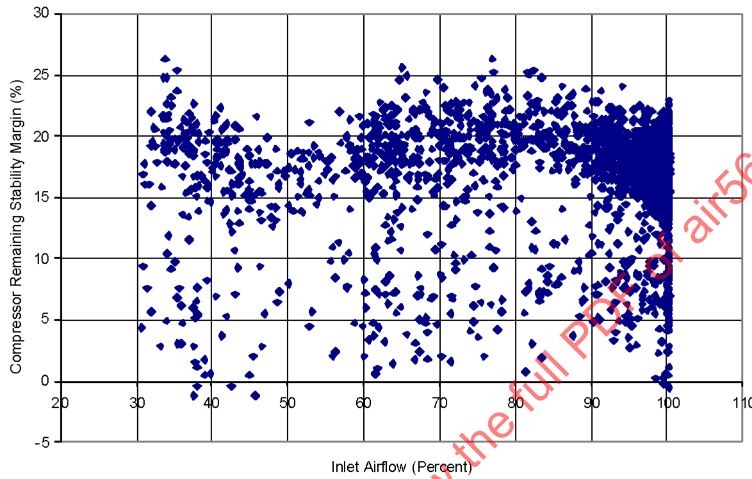


FIGURE 46 - PREDICTED COMPRESSOR STABILITY MARGIN REMAINING

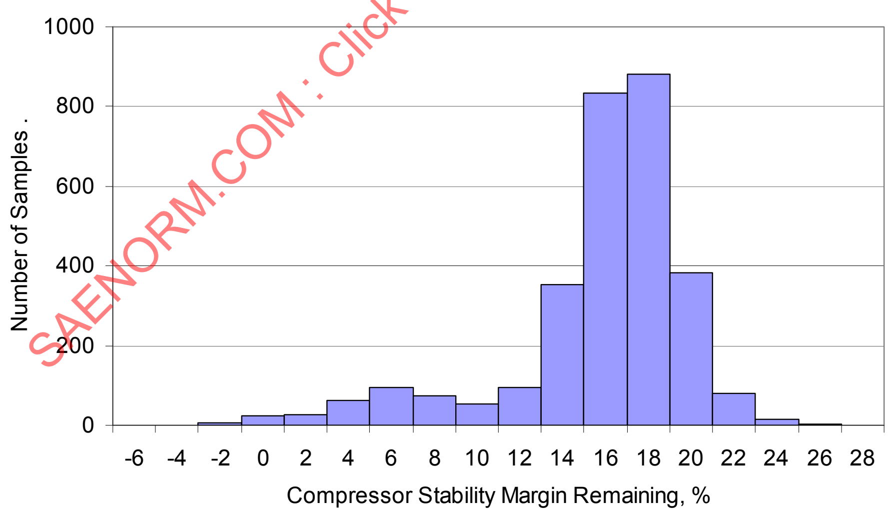


FIGURE 47 - PREDICTED COMPRESSOR STABILITY MARGIN REMAINING HISTOGRAM

6.4 Determination of Stall Rate

While the stall probability based on the number of total engine events is useful, it is more desirable to have a stall rate based on engine flight time and usage. With the availability of usage data similar to Figure 8 from a fleet of engines, the numbers of transients of each type in each Mach/altitude box are available for a certain number of engine flight hours. Seeing that throttle transients are a major factor in stall margin accounting, it becomes an obvious choice to equate throttle usage to a stall rate prediction. Since the transient type selection logic in the model represented in Figure 15 is based on the same usage data, a certain number of transients from each Mach/altitude block, as modeled in the simulation, can be equated to an actual known engine flight time. It is then possible to determine the stall rate per engine flight hour since a certain sample size is equated to a certain engine flight time. For the determination of stall rate, it is essential to have the distributions in each Mach/altitude box be based on percentage of total transients rather than percentage of total time. This requirement is based on the necessity to equate a certain number of transients with a certain known flight time. The different transients have different times associated with their occurrences. One method for implementing this process is to place “counters” on a certain number of the transient types in specific Mach/altitude boxes. This “counter” is a simple algorithm that counts the number and type of transients that occurs in a specified Mach/altitude box. When the simulation run is complete, the number of transients counted divided by the number of transients for a specific known flight time can be used to compute the stall rate per engine flight hour. For example, if a Mach/altitude box had 800 A/B transients in 1000 flight hours, then when the counter reached 800 A/B transients, that would equate to 1000 flight hours (or if fewer or more are counted, then the ratio would be maintained). For the example model, the top ten usage boxes from the Mach/altitude envelope were chosen and counters were placed on the top three highest usage transients in those boxes for a total of thirty counters. The flight time and stall rates were averaged for this set of thirty counters for each simulation run.

6.4.1 Sample Size for Stall Rate Determination

The next step is to determine the sample size that will produce good repeatability of the model. In the field, an aircraft could operate for hundreds of hours without an engine stall. However, that same aircraft could have multiple stalls within the next few flights. Since the example model was built to replicate the field operations, this same “randomness” was built into the model. As an example, for a certain sample size, successive runs can vary significantly in the number of stalls computed. The main reason for this variance is the fact that near stalls (near zero stability margin remaining) are not considered. When the average remaining fan stability margin was considered, the result converged so that $\pm 2\sigma$ was within ±2% with a data set size of 3000. However due to the random nature of stalls, neither the stall rate nor the stall probability will converge within these criteria. Additionally, the determination of stall rate has an added parameter of time. The equating of the number of transients to a certain time period incorporates the need for additional points since there is added variability in the time domain.

Due to this variability in the occurrence of stalls, it is difficult to determine the sample size necessary for good repeatability. Each simulation will be different in the number of points needed for good repeatability and, therefore, it is advisable to check out the repeatability of each simulation similar to the checkout of the example simulation. Good repeatability for the example case was determined to be adequate when both the average stall rate and the standard deviation of the data sets were not significantly changed. For the example simulation, sample sizes of 1500, 3000, 9000, 18 000, and 36 000 were run. The results for the example simulation are shown in Figure 48 and Figure 49. (It is advisable to run 33 cases at each sample size in order to get a good statistical distribution and to check that the periodicity of the random number generator is greater than twice the sequence required as suggested in Appendix A.2.) For our notional aircraft simulation, it was determined that a sample size of 18 000 was adequate for good repeatability.

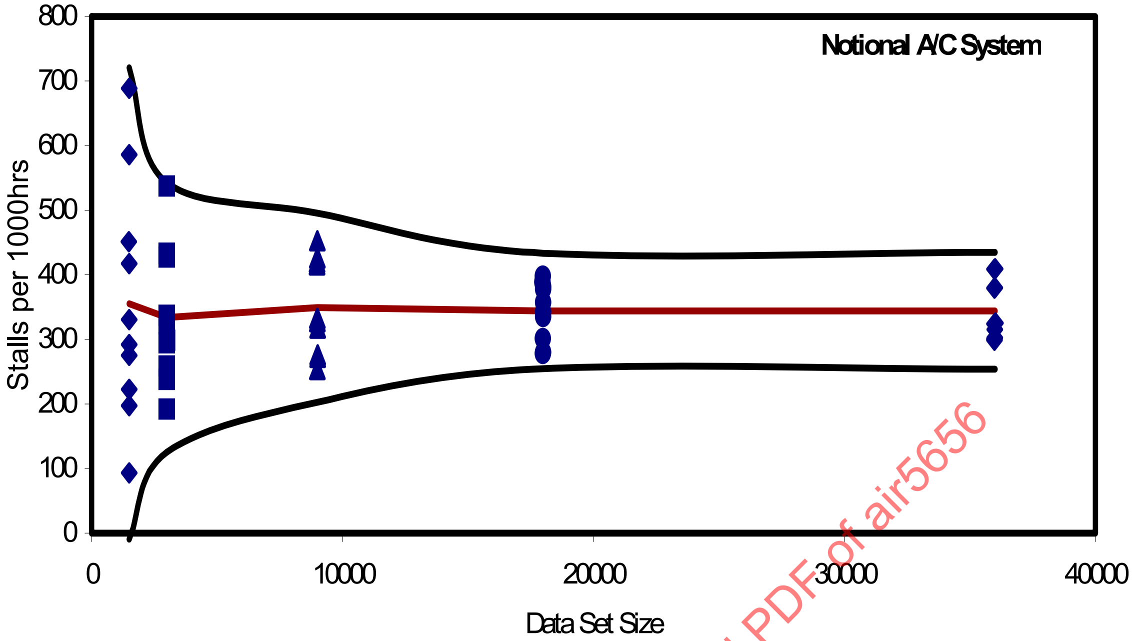


FIGURE 48 - STALL RATE CONVERGENCE WITH INCREASED SAMPLE SIZE

Sample Size	Average stall rate per 1000 EFH	Standard Deviation	+/- 5% of avg.	+/-10% of avg.	+/-2σ
1500	355.54	182.48	373.32	391.09	720.51
			337.76	319.99	-9.42
3000	334.11	104.35	350.82	367.53	542.81
			317.41	300.70	125.42
9000	349.18	73.20	366.64	384.09	495.58
			331.72	314.26	202.77
18000	344.14	44.67	361.35	378.56	433.49
			326.94	309.73	254.80
36000	344.31	45.02	361.52	378.74	434.35
			327.09	309.88	254.27

FIGURE 49 - STATISTICAL ANALYSIS OF STALL RATE CONVERGENCE WITH INCREASED SAMPLE SIZE

However, with the results of the notional aircraft system simulation, the stall rate was significantly higher than would be expected of an operational aircraft. The notional simulation had adders and multipliers placed on it to decrease the stability margin and to increase the distortion. These adders and multipliers served to exaggerate the number of stalls in order to evaluate the sensitivity of the simulation to input parameters. Even though these modifiers only resulted in a stall probability (stalls per event) of 0.5%, the resulting stall rate based on the usage simulation was over 300 stalls per 1000 EFH (which is an unrealistic scenario).

6.4.2 Realistic Aircraft System

A more realistic fielded stall rate for a marginal aircraft system would fall in the range of 3 to 10 stalls/1000 EFH (fighter application). Because stalls would be significantly less frequent than our original notional aircraft system simulation, the tails of the distributions would have to be adequately sampled and therefore potentially more cases would be necessary for good repeatability. For this reason a more realistic marginal system had to be constructed with reduced adders and multipliers on the stall lines and distortion sensitivities to change the stall margin and the distortion consumption of stall pressure ratio. This model will be called a “realistic aircraft system”. In order to get repeatability with this more realistic system, significantly more points were run since far fewer stalls were being computed. Sample sizes of 36 000, 100 000, 200 000, 400 000, and 800 000 points were run for the example simulation. The following figures (Figures 50 and 51) show the results from the example cases. From these results, it was determined that for the example model, a sample size of 400 000 was appropriate for good repeatability. By 400 000 points, both the average stall rate and the standard deviation of the data set are reasonably stable, and good repeatability for this simulation was obtained.

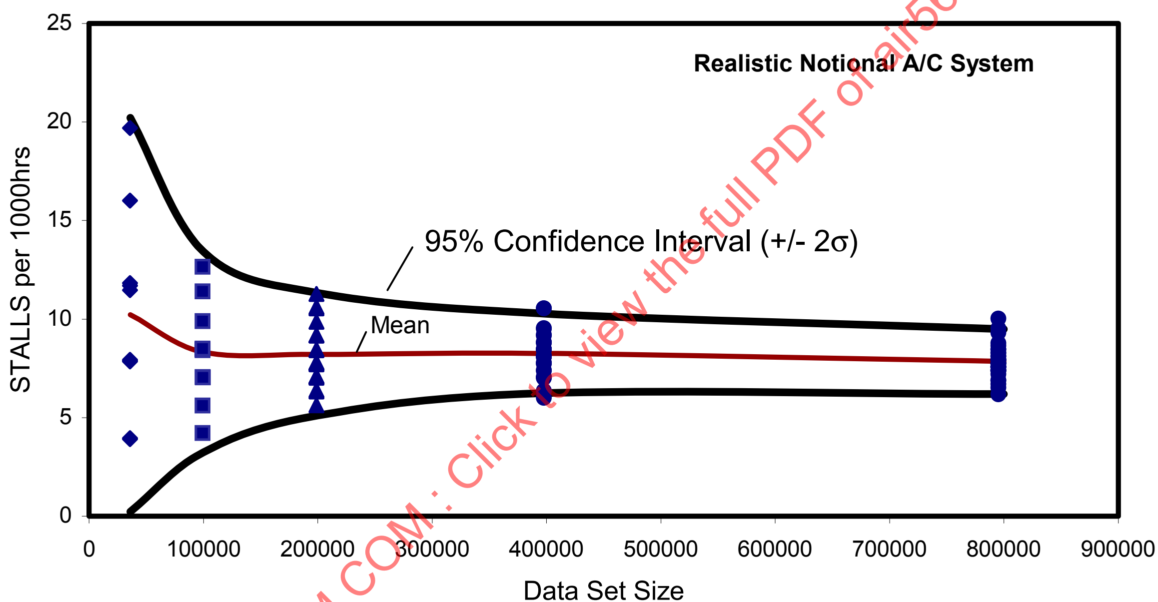


FIGURE 50 - STALL RATE CONVERGENCE WITH INCREASED SAMPLE SIZE FOR REALISTIC SYSTEM

Sample Size	Average stall rate per 1000 EFH	Standard Deviation	+/- 5% of avg.	+/-10% of avg.	+/-2σ
36 000	10.22	5.00	10.73	11.25	20.22
			9.71	9.20	0.23
100 000	8.33	2.54	8.74	9.16	13.41
			7.91	7.49	3.24
200 000	8.22	1.55	8.63	9.04	11.32
			7.80	7.39	5.11
400 000	8.25	1.00	8.67	9.08	10.26
			7.84	7.43	6.25
800 000	7.85	0.82	8.24	8.63	9.50
			7.45	7.06	6.20

FIGURE 51 - STATISTICAL ANALYSIS OF STALL RATE CONVERGENCE FOR REALISTIC SYSTEM

6.5 Trade Studies and Sensitivity Analysis

Once a simulation has been validated, system trade studies and the relative merits of modifications can be evaluated simply as a change in number of stalls per event. In this section, the statistical stability assessment methodology is applied to the notional aircraft system that was described in 6.3 and the predicted results of two or more sets of input variables or distributions are compared. Also included are some SSA case studies from actual in-service systems. It should also be recognized that a sample size of 3000 was again used in the following studies/analysis as discussed in 6.2.2.

6.5.1 Environmental Trade Studies

Ambient temperature represents one of the most common environmental variables. Statistical data to model ambient temperature variation are readily available and are discussed in 5.1. A trade study that predicts the number of stalls per thousand events for both the fan and engine compressor is illustrated in the following example. The sensitivity to the ambient temperature was interrogated by running the simulation, with a sample size of 3000 for MIL-STD-210A hot, standard, and cold atmospheres. The impact that ambient temperature has on the stall rate is shown in Figure 52, where an increasing stall rate with decreasing temperature is predicted for both the fan and compressor. Also, the fan has a greater sensitivity to ambient temperature change than the compressor. As ambient temperature decreases, the engine is able to maintain maximum airflow further to the right in the altitude/Mach number envelope, increasing the exposure of the engine to higher distortion levels. The reverse is the case as ambient temperature increases. The fan is more sensitive than the compressor because it has a greater rate of change of available stability margin with airflow than does the compressor.

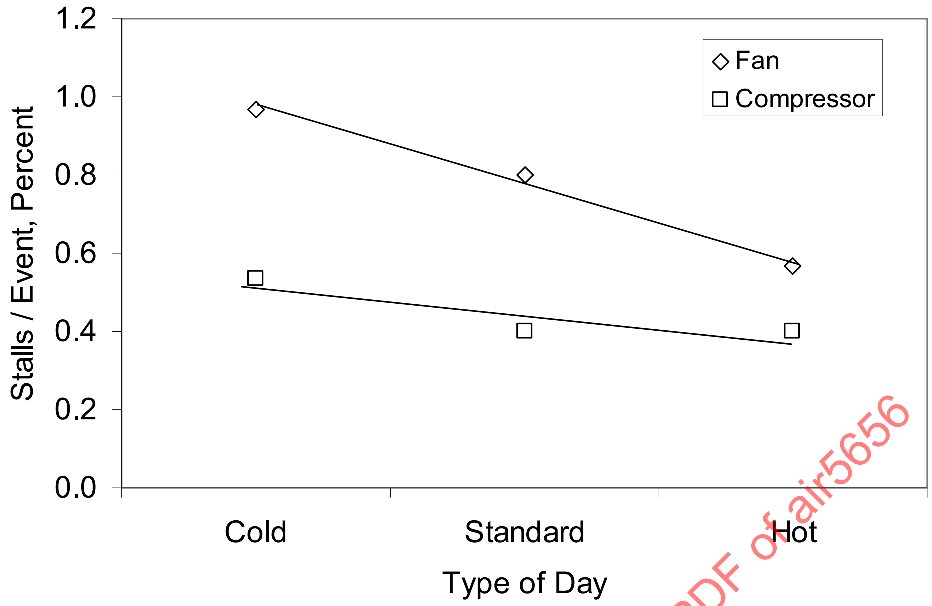


FIGURE 52 - TEMPERATURE SENSITIVITY

6.5.1.1 Theatre Specific Application

The following example is based on the application of a Statistical Stability Assessment to an in-service weapon system. A squadron of aircraft assigned to a specific base is considered in the example where actual weather conditions are much colder than the standard atmosphere. A comparison of the sea level worldwide standard temperature distribution and the distribution for an airfield in a cold weather environment was shown in Figure 6.

The SSA simulation specifically added compressor blade tip clearance calculations modeled as a function of rotor speed, case and rotor temperature, and the case pressure delta. This higher-fidelity model provided greater insight into the causes of stall events. As illustrated in Figure 53, the remaining stability margin for operation in an extreme cold weather environment was significantly less than that predicted for operation in a standard atmosphere. The corresponding predicted loss in compressor stability margin, due to compressor blade tip clearance, is illustrated in Figure 54, and is clearly related to the variation in ambient temperature. Analysis of these results led to an understanding of the slight increase in engine stall rate that was occurring at this base and led to a simple engine control system revision that eliminated the increase.

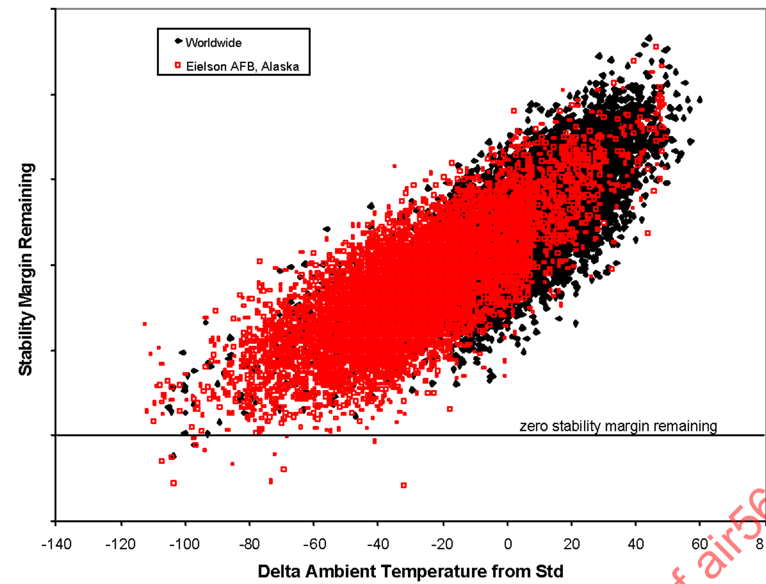


FIGURE 53 - AMBIENT TEMPERATURE EFFECTS ON STABILITY MARGIN

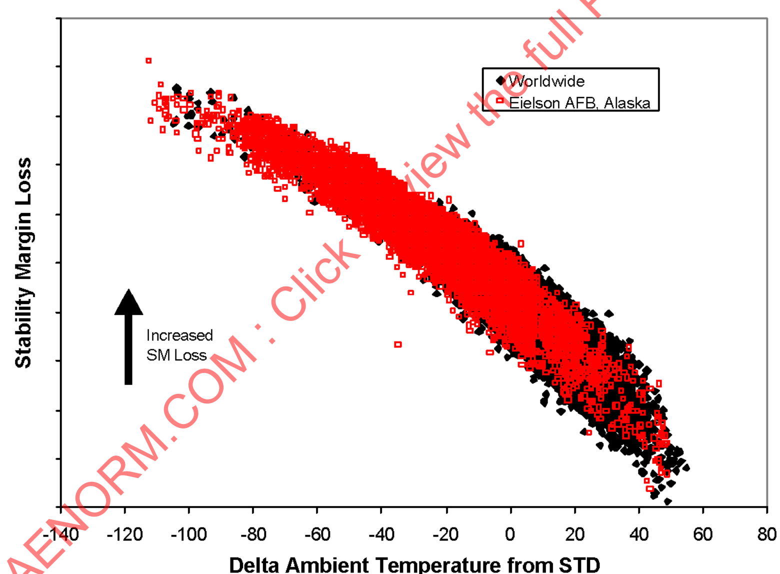


FIGURE 54 - EFFECTS OF AMBIENT TEMPERATURE ON STABILITY MARGIN

6.5.2 Aircraft Usage Trade Studies

Aircraft usage trade studies refer to variations in the primary variables of aircraft usage such as Mach number, altitude, AOA, AOSS, and throttle usage. Changes in these parameters can be caused by a change in the type of mission assigned to a specific group of aircraft or a change in the mission mix.

The effect that a change in aircraft mission may have on engine stability can be studied by observing the predicted stall rate in different regions of the flight envelope. A statistical stability assessment was performed over the whole envelope and at specific altitude/Mach number regions. The corresponding predicted stall rates are shown in Figure 55. The upper region of the envelope was 35 to 45kft and 0.4 to 0.8 Mach number. The middle region of the envelope was 15 to 25kft and 0.4 to 0.8 Mach number and the lower region was 0 to 10kft and 0.4 to 1.0 Mach number.

In the middle region of the envelope, the predicted fan stall probability is consistent with the overall rate. However, the compressor-stall probability increased to 2.33%. Off-idle stalls drive the increase in compressor stall rate because full throttle excursions occur more often in this region. Flight idle is at low power in this region because no control limits (such as minimum burner pressure) are encountered.

In the upper region, the predicted stall probabilities are approximately 19% and 5% for the fan and compressor, respectively. Since this region has relatively low time exposure, these rates do not significantly affect the overall rate. Clearly, if the mission time in this region were increased, the whole envelope engine stall rate would be expected to increase significantly.

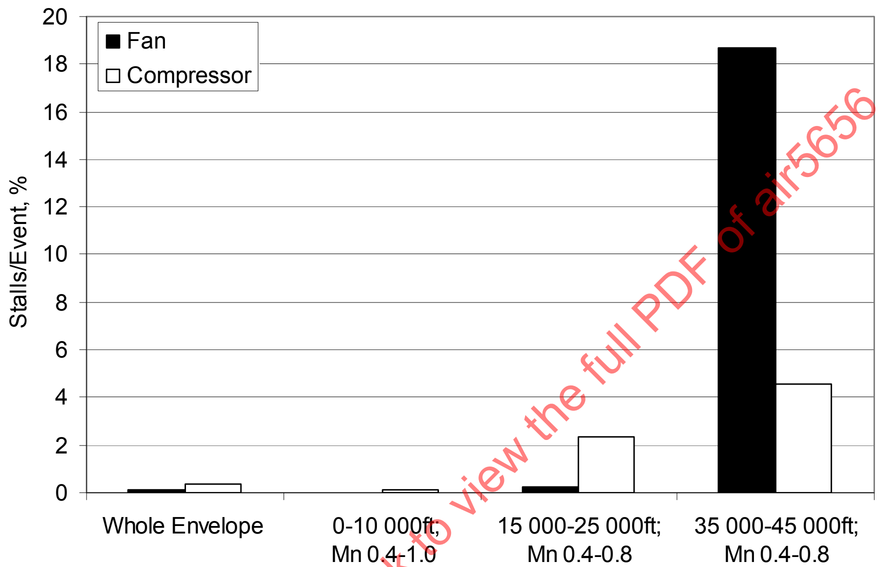


FIGURE 55 - PREDICTED STALL RATE BY MISSION FLIGHT REGION

The lower region shows a very low stall rate, as expected, since most of the destabilizing influences were minimized. Distortion is limited due to the restricted maneuver envelope and both augmentor and gas generator transient pressure spikes are greatly reduced.

The significantly different stall rates predicted for the different flight-envelope regions illustrate the importance of the aircraft mission usage in determining the overall stall rate.

6.5.3 Aircraft Installation Trade Studies

Aircraft installation trade studies refer to variations in the aircraft installation demands such as inlet distortion, and engine bleed and power off-take demands. Airframe inlet modifications or additional electrical equipment can also cause changes in these parameters.

6.5.3.1 Power Off-Take Variation

A nominal power off-take of 75 horsepower was used for a baseline analysis. To investigate the effect of power off-take on stall rate, a distribution of power off-take usage was obtained from overall hydraulic system statistical analysis. The simulation-input distribution is presented in Figure 56. As an alternate case, a dependency between horsepower off-take and angle of attack was postulated based on the idea that the control surfaces must move to achieve non-cruise angle-of-attack conditions. The effect of hydraulic accumulators should be considered when determining the dependency between control surface movement and power off-take. The resulting predicted output distribution is shown in Figure 57. The predicted stall rate for the baseline, non-dependent distribution, and the dependent distributions are compared in Figure 58. The non-dependent distribution shows a modest increase in stall rate, but the dependent distribution shows a more significant increase in stall rate. This example demonstrates the importance of understanding and modeling variable dependencies.

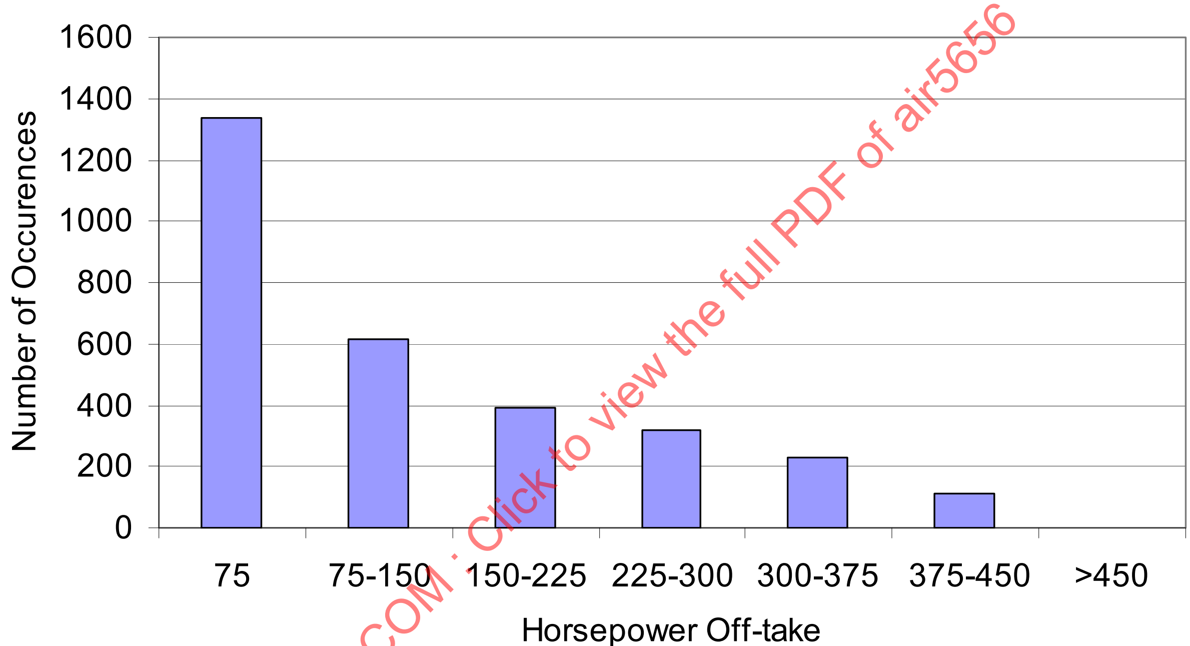


FIGURE 56 - NON-DEPENDENT HORSEPOWER OFF-TAKE DISTRIBUTION INPUT

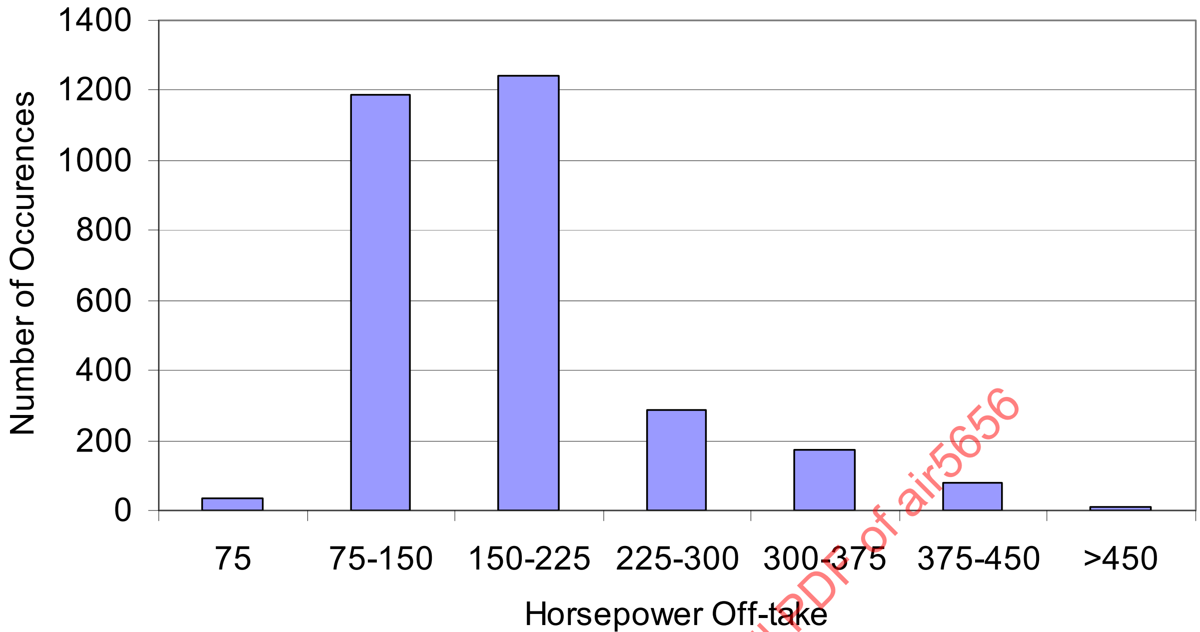


FIGURE 57 - PREDICTED OUTPUT HORSEPOWER OFF-TAKE DISTRIBUTION
(DEPENDENCY ON ANGLE OF ATTACK)

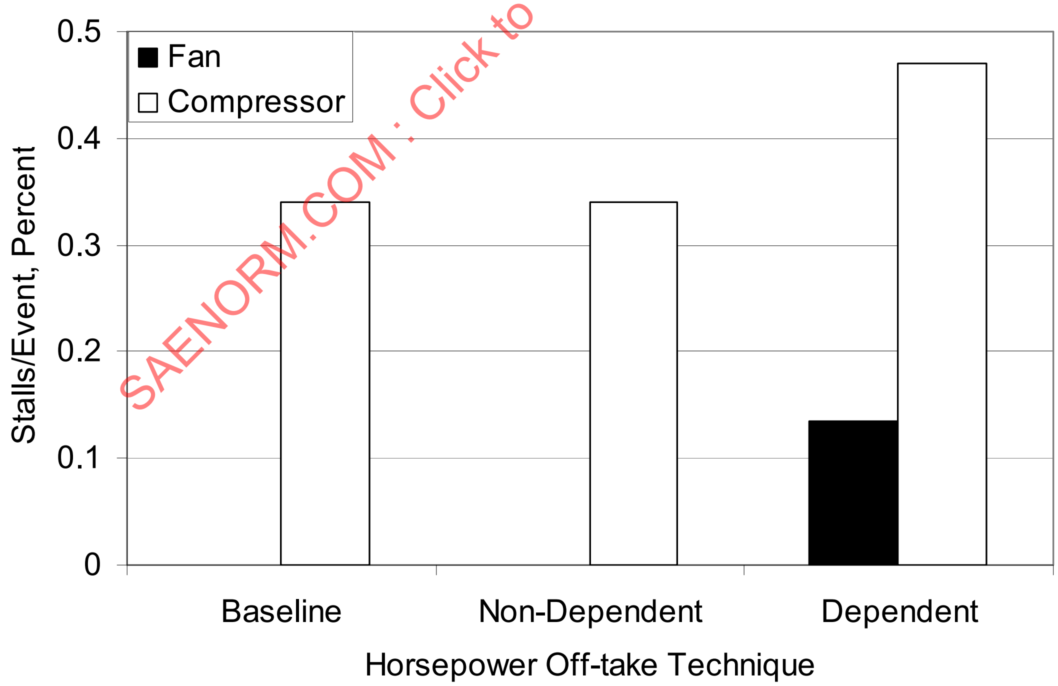


FIGURE 58 - PREDICTED STALL RATE WITH VARYING HORSEPOWER OFF-TAKE DISTRIBUTIONS

6.5.3.2 Airframe Modifications

Airframe modifications that affect the inlet or forebody can induce changes to the engine inlet distortion levels. The effect that increased circumferential distortion could have on predicted engine stall rate is presented in Figure 59. As expected, when the circumferential distortion is increased, the number of stalls also increased. Results such as these could then be used in a trade study to decide if a particular level of change is acceptable.

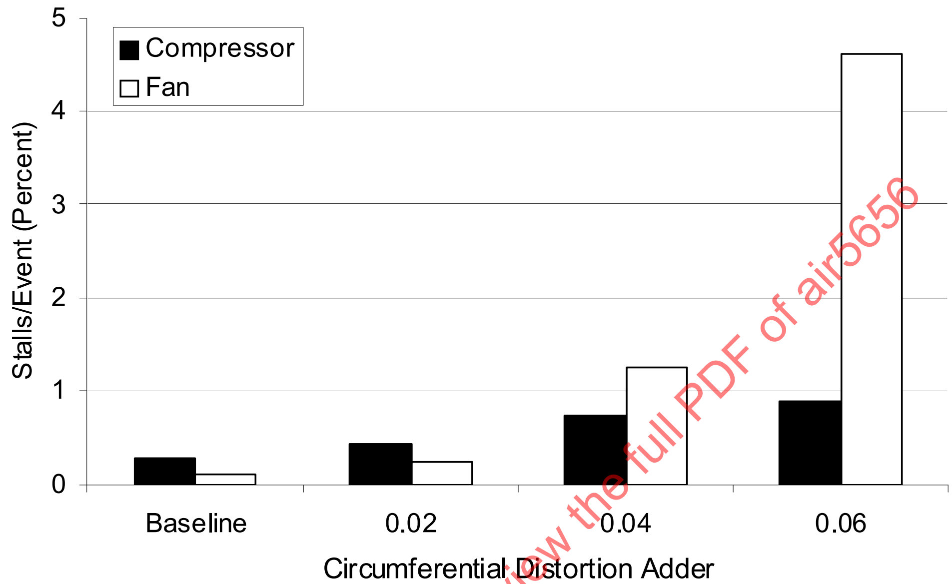


FIGURE 59 - PREDICTED EFFECTS OF INCREASED DISTORTION

6.5.4 Engine Trade Studies

Engine trade studies can be performed to explore the impacts of variations in engine component characteristics. The studies can be used to evaluate a wide range of variations such as changes in control logic, different operating schedules, and blade-tip clearances. Another example would be the comparison of two different engines in the same aircraft installation.

6.5.4.1 Engine Control Schedule Variations

Stability characteristics of gas turbine engines are greatly influenced by control systems. Typical controlled variables include rotor speeds, pressure ratios, fuel flows, and geometric parameters such as angles and areas.

An example of the use of the SSA to determine the effects of a change in the engine control to enhance the stability margin of a component is illustrated in Figure 60. In this case the variable inlet guide vanes to the compressor were rescheduled to reduce the compressor sensitivity to blade tip clearances as a function of the operating environment. Logic change I represented an engineering change that had already been approved. Logic changes II and III were proposed engineering changes that represented progressively more aggressive vane rescheduling. Logic change III also included a time constant on the most aggressive scheduling of the vanes, biased by ambient temperature to address cold day take-off scenarios only. The impact on HPC stall rate throughout the life of the engine is shown in Figure 60.

This use of the SSA greatly aided the decision process by quantifying the risk versus cost of implementing the proposed engineering changes.

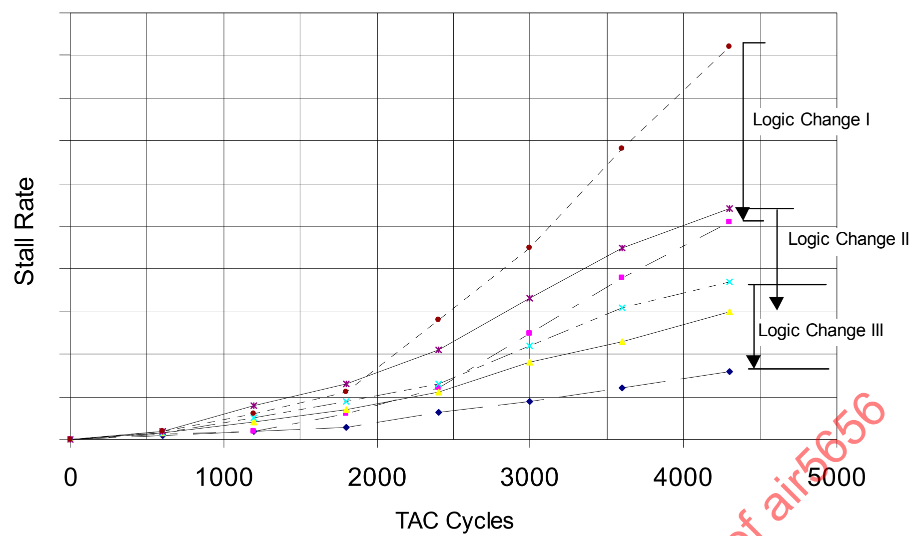


FIGURE 60 - EFFECT OF CONTROL LOGIC ON STALL RATE FOR AGED FIGHTER ENGINE FLEET

6.5.4.2 Engine Component Modifications

To perform an engine component trade study, the fan map was altered in the engine model. The results of using the simulation to predict the reduction in fan stability margin due to a durability-enhancing fan modification are shown in Figure 61. The stall rate for the baseline fan stability margin was acceptable at approximately 0.2%. However, when the margin was reduced by four percent, the predicted stall rate increased to 20% (off scale on figure). A one-percent increase in stability margin resulted in a prediction of no stalls.

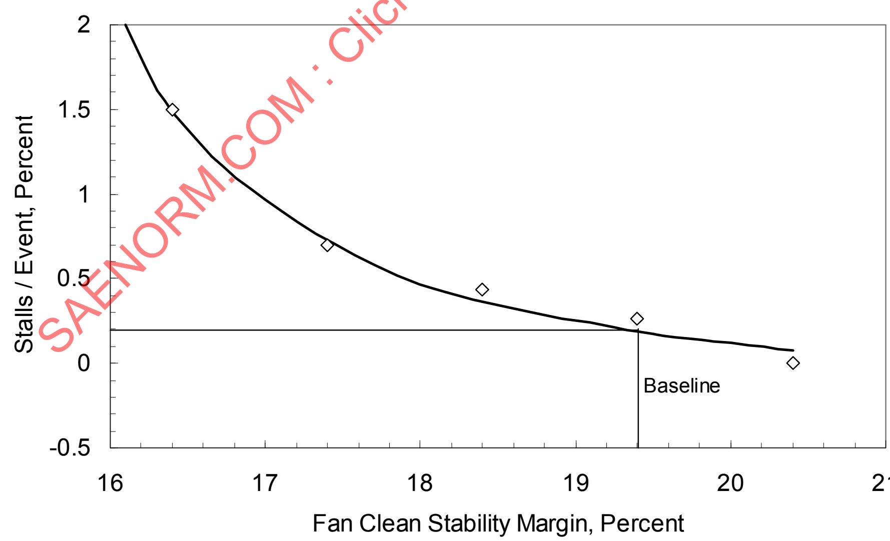


FIGURE 61 - EFFECT OF FAN STABILITY MARGIN ON PREDICTED STALL RATE

A similar study was performed for the HPC, with results as shown in Figure 62. In this study, the impact of an increase in the HPC operating line (intended to increase thrust and decrease specific fuel consumption) was simulated. With a baseline of 22% stability margin, a three percent reduction in stability margin increased the predicted stall rate difference by approximately 1.5%.

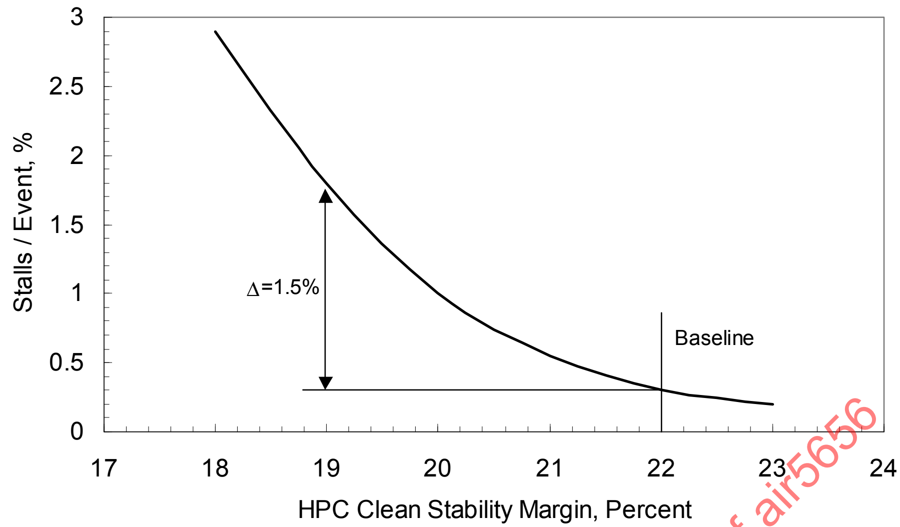


FIGURE 62 - EFFECT OF COMPRESSOR STABILITY MARGIN ON PREDICTED STALL RATE

6.5.4.3 Fleet Aging Trade Study

A sensitivity analysis of fleet-engine aging was performed using the statistical stability assessment. Deterioration in the engine model was defined as a function of Total Accumulated Cycles (TAC). The TAC value was varied from zero, representing a new engine, to 4000 representing the scheduled overhaul cycle. The study results are shown in Figure 63. In the case of the compressor, it is interesting to note that the predicted stall rate peaks at about 2000 cycles and then decreases steadily to the 4000 TAC overall cycle. This occurs because the high-pressure turbine area continues to increase with age from the 2000 to 4000 TACs interval. Further, because the turbine temperature limit is encountered near the 2000 TAC peak, the compressor match point at high power is rolled back into an area of higher available stability margin. The fan stall rate steadily increases and then also approaches the peak level at the 2000 TACs point, due to increased tail pipe temperature, which rematches the fan to a higher pressure ratio. Once the cycle temperature limit is encountered (~2000 TACs), the tail pipe temperature increase ceases and the stall rate flattens.

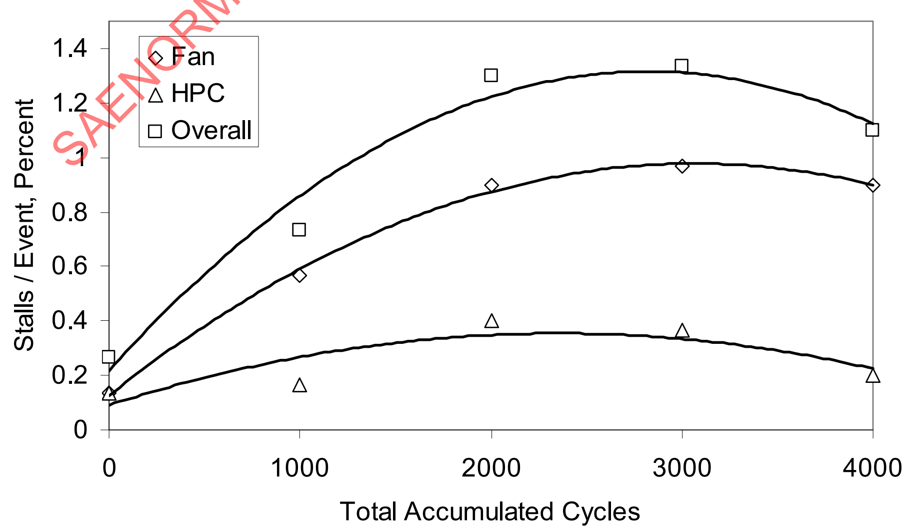


FIGURE 63 - EFFECT OF FLEET AGING ON PREDICTED STALL RATE

As the fleet ages there will, in actuality, be an engine age mix or distribution, as discussed in 5.4.3.3. In this section, two different age distributions were analyzed and the results are compared with a baseline, new engine profile, in Figures 64 and 65. In the first case, the selected distribution represented a fairly young fleet with about 65% of the engines at 2000 TACs or less. As expected, both the fan and compressor show an increase in predicted stall events as the fleet ages. On a percentage basis the stall rate increase is quite manageable and therefore, no operational problems would be expected. In Figure 65, the age distribution was selected to be representative of a more mature fleet with over 80% of the fleet at 2000 TACs or greater (see distribution). The fan stall rate increased slightly over the younger distribution, and the compressor stall rate actually decreased because of the engine rematch discussed previously. These results are consistent with the sensitivity study shown in Figure 63.

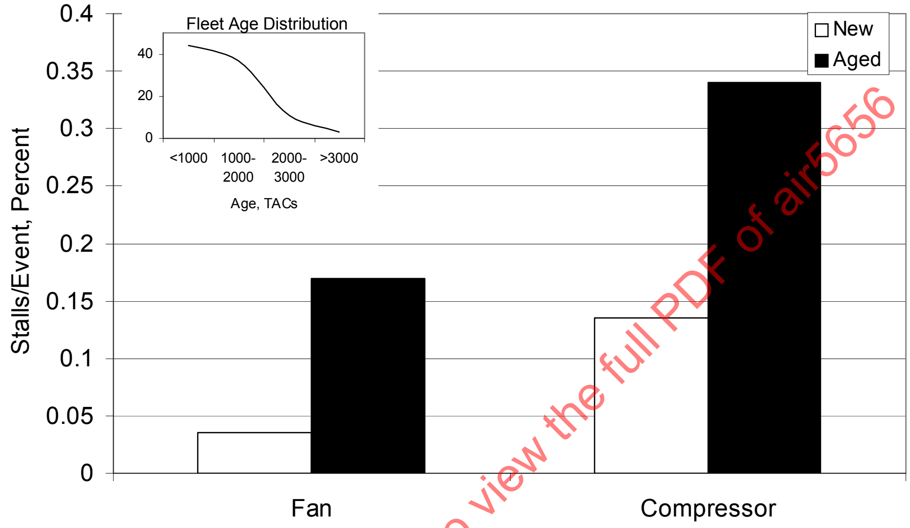


FIGURE 64 - AGE DISTRIBUTION AND STALL EVENTS OF YOUNG FLEET

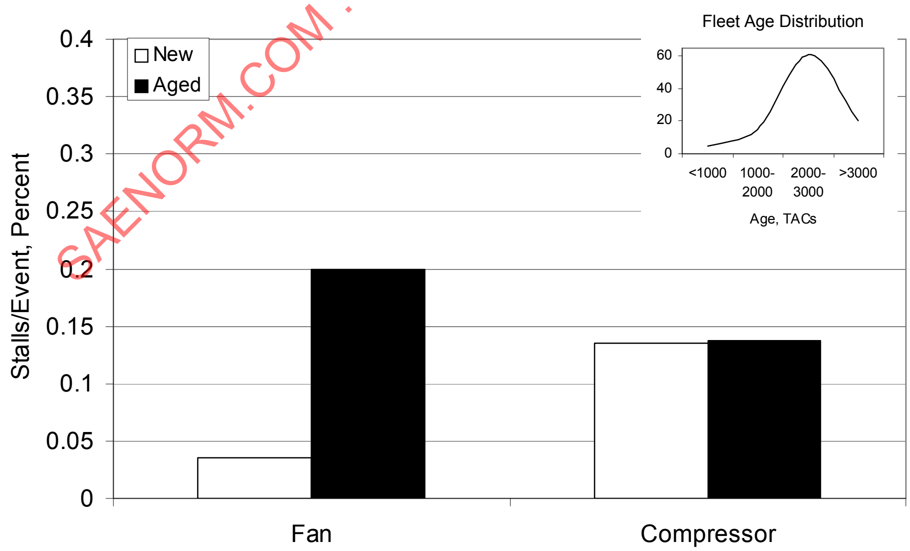


FIGURE 65 - AGE DISTRIBUTION AND STALL EVENTS OF MATURE FLEET

This type of trade study could be conducted even before the engine enters service. Based on this analysis, stalls or operability problems could be avoided between planned overhauls.

7. VALIDATION OF SSA

Validation is an important consideration prior to the use of any simulation. In the case of simulations involving extreme-value statistics, as is the case with the Monte Carlo based SSA, validation is a recognized problem. The validation process should be performed at a number of levels. These should cover a fundamental check of the calculations performed by the simulation, validation of input/output data transfer between routines, a check of the individual statistical distribution from each of the parameter distributions, and validation of the overall simulation against current practices and service data, where available.

Of particular significance to the SSA, where the stall rate may be dependent on extreme values, the sampling of the tails of the distributions requires close scrutiny. The number of samples used in the simulation can also have a significant impact on the confidence of the output. The simulation should be performed with a range of sample sizes to determine the sample size necessary to achieve a repeatable result with adequate confidence. This was discussed in 6.2.2 and is further discussed in Appendix A.3.

Guidance on the validation of the simulation against current-practice audits (see Appendix B and AIR1419) and service data is provided in the following sections.

7.1 Validation Against Current Methodology

An important consideration in the validation process is the ability to replicate the current-practice audit assessed at a single design or flight condition. Within a given company's audit methodology, this may be established at a -2σ or -3σ point for random factors that are assumed in the audit. The individual random factors contained within the current practice stability audit are usually obtained from data with the variation defining the values of σ. These random elements are then root-sum-squared (RSS) in the audit to give statistical recognition that their extremes are unlikely to occur simultaneously. This RSS value is the statistical σ value in the overall audit process. The non-random factors are directly summed and represent the average engine.

To test the replication of the above audit, it is necessary to set a default option within the SSA simulation that automatically samples the distribution of each destabilizing factor at the flight and operating condition equivalent to the classical audit. The random elements contained within the engine audit would still sample their entire distribution to replicate the random portion of the audit. The SSA simulation would then be run with a sufficiently large sample size to adequately cover the random element distributions. A typical comparison between the current practice stability audit and the default option mode of the SSA simulation is shown in Figure 66.

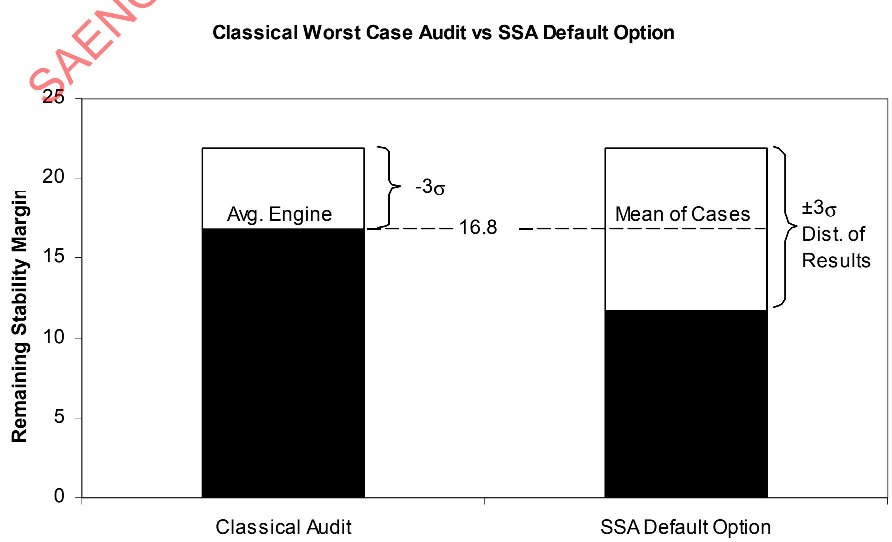


FIGURE 66 - COMPARISON OF CLASSICAL AUDIT TO SSA DEFAULT OPTION

This default option should be checked (as a minimum) at all the key operability design or pinch point conditions. This validation process, when complete, verifies that the distributions have been entered correctly, and the basic audit methodology and the associated mathematics have been correctly constructed within the SSA simulation. In this example, the SSA would be successfully validated relative to the classical audit. The mean of the SSA results is equal to the sum of the non-random (average engine) from the classical audit, and the σ from the SSA resulting distribution equals the RSS derived σ from the classical audit.

7.2 Comparison With Service Experience

A final step in the validation process for the SSA simulation is to check the prediction of stall rate against a known fielded system that has a statistical database from a reliable tracking system. Since the mid-1970s, many military and commercial systems have flight data recorders and engine monitoring systems that can track the necessary statistical data.

The overall stall rate of a particular weapon system is typically plotted as a rate against the accumulated engine flying hours. The examples contained within 6.5 presented the basic output from the simulation in terms of stalls encountered for a given number of samples. In order to determine a stall rate, it is necessary to establish a relationship between the number of samples and engine flying hours. A method to achieve this was presented in 6.4.

An accurate prediction of stall rate provides good evidence that a valid simulation has been achieved but care should be taken that the result is achieved through accurate modeling of the piece parts, as described in the above sections, and is not just fortuitous. Once a simulation has been validated against the in service stall rate, system trade studies and the relative merits of modifications can be evaluated simply as a change in number of stalls per sample.

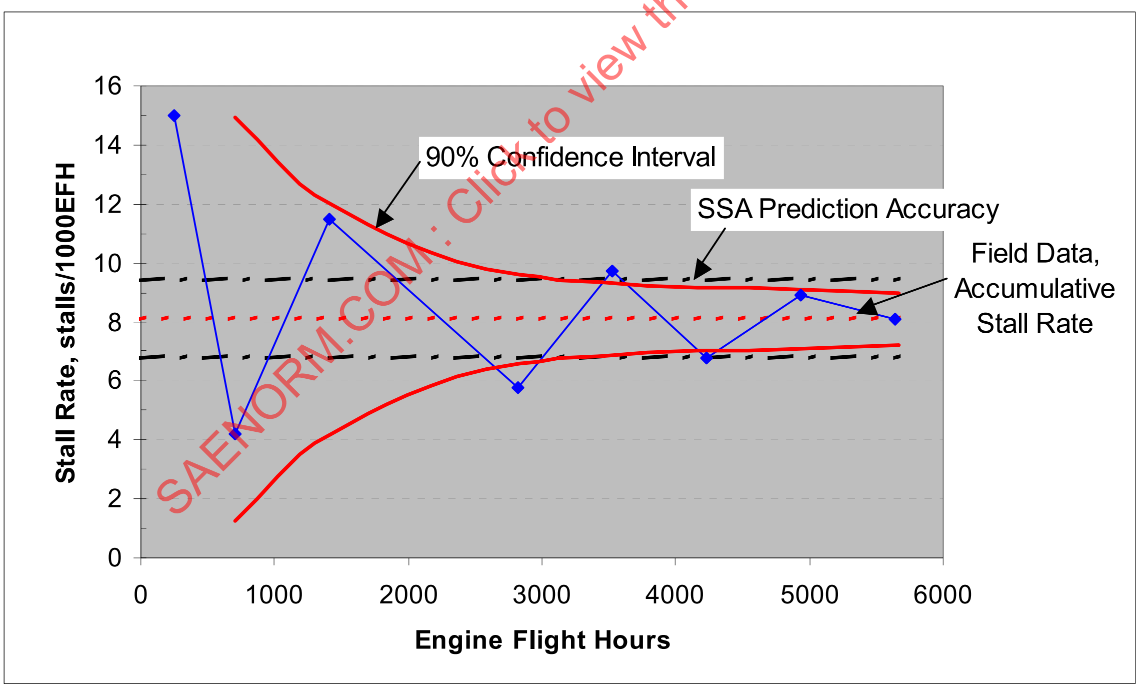


FIGURE 67 - ACCUMULATIVE STALL RATE OF TYPICAL FIELDED SYSTEM

A comparison of converged stall rate from a simulation, such as described in Figure 50, with notional field data is shown in Figure 67. Although no standards have been established for the accuracy of SSA model predictions of stall rate, a reasonable accuracy would be within the 90% confidence interval of the systems field data. A typical 90% confidence interval for a chi-square reliability rate for the data is displayed in Figure 67. The ±1.5 stall rate accuracy band brackets this confidence interval as the systems approaches a mature life (approximately 500K flight hours for this example). The SSA simulation can be “tuned” to improve the accuracy, as additional field and production data become available.